The TE_{00} Waveguide Mode – The "Complete" Story

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Abstract

A partial eigenfunction expansion of the electric-type dyadic Green's function used in aperture-coupled waveguide problems is discussed in connection with the traditional Green's function expansion in terms of the waveguide modes. Based on the principles of distributions, the delta-function term is extracted from a double series, resulting in the complete representation of the Green's function in the source region. This, in turn, is related to inclusion of the term with zero indices in the computation of the double-series expansion, even though it does not correspond to any waveguide mode. The effect of exclusion of this term from the series, and controversies over published results in the analysis of slotted-waveguide couplers and radiators, are illustrated.

Keywords: Green function; vectors; waveguides; rectangular waveguides; cylindrical waveguides; waveguide antennas; waveguide couplers; waveguide excitation; waveguide theory; eigenvalues and eigenfunctions; slotted waveguides; source-point singularity

1. Introduction

A complete representation of the electric dyadic Green's function of cylindrical waveguides and cavities has been actively discussed over the past few decades, resulting in a good understanding of the singular behavior of the Green's function in the source region. Nevertheless, there are controversies concerning published results for the analysis of slotted waveguides (particularly, for longitudinal and tilted slot coupling) that employed a partial eigenfunction expansion of the electric dyadic Green's function. This paper is intended to clarify this disagreement by providing insight into the correlation between different forms of the Green's function.

Below we summarize some previous work on this topic. In [1], the expansion of the electric dyadic Green's function in terms of solenoidal eigenfunctions [2] for the electric current source in a rectangular waveguide was revisited by adding a delta-function term to make this expansion complete in the source region. This was generalized in [3] for a rectangular cavity for different representations and types (magnetic vector potential and electric) Green's dyadics. In [4], an alternative approach, using the theory of distributions, was proposed to obtain a complete representation of electric dyadic Green's functions for rectangular waveguides and cavities. It was based on the solution of a vector potential Green's dyadic and the relation between electric and magnetic Green's dyadics. In [5], general expressions for the complete expansions of electric and magnetic Green's dyadics in the source region were obtained in terms of solenoidal eigenfunctions with an additional delta-function term introduced for the electric Green's dyadic. Also, complete expansions of different dyadic Green's functions for cylindrical waveguides were presented in [6], emphasizing the presence of the delta-function term. In [7], electric and magnetic Green's dyadics were given in connection with the scattering from discontinuities in a hollow waveguide. The electric Green's functions discussed above were obtained to represent the electric field due to an electric-current source in the presence of a perfectly conducting boundary (waveguide, cavity). A dual problem for the magnetic field due to a magnetic current involves the magnetic dyadic Green's function (or electric Green's dyadic of the second kind), wherein the delta-function term is introduced to complete the expansion of solenoidal eigenfunctions in the source region [8].

The above work is related to the analysis of the electric field in the source region within an unbounded domain using a principal-volume integration of the free-space electric dyadic Green's function [9-13], where different principal-volume geometries (pill-box, sphere, slice, etc.) can be used. Also, it is important to note

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that the delta-function term in the electric dyadic Green's function for waveguides is not a singularity of the Green's function, but instead corresponds to its irrotational part. The remaining solenoidal-eigenfunction-expansion part of the Green's function is highly singular in the source region [14, 15]. This was also pointed out in [12] by comparing different forms of the electric Green's dyadic for the rectangular cavity.

An alternative representation of the electric (magnetic) dyadic Green's function for closed-boundary cylindrical waveguides and cavities, due to the electric (magnetic) current source, is the partial eigenfunction expansion [16-18]. In this case, the Green's function is obtained as a series expansion over the complete (in $\mathbf{L}^2(\Omega)$, where Ω represents the waveguide's cross section) system of eigenfunctions of a self-adjoint Sturm-Liouville operator (particularly, for cylindrical waveguides and cavities, the transverse Laplacian operator), with the one-dimensional characteristic Green's functions in the direction of propagation. This type of expansion has been applied in [19] in the theory of slotted rectangular waveguides, and later in the analysis of cylindrical tubes [20, p. 301, and references therein] and waveguide discontinuities [21]. In [4], this expansion was used in the derivation of the vector-potential Green's dyadic for rectangular waveguides and cavities. Electric dyadic Green's functions for a multilayered rectangular waveguide were developed in [22, 23] in the analysis of shielded microstrip-like transmission lines. In [24 and references therein], the partial-eigenfunction-expansion method was applied to obtain electric dyadic Green's functions of the first and second kind for a transversely layered rectangular waveguide, with applications to waveguide-based aperture-coupled patch arrays used in spatial power combiners.

It should be noted that the partial eigenfunction expansion of the electric (magnetic) Green's dyadic does not contain the delta-function term separately, but it does represent a complete form of the Green's function in the source region. Regarding rectangular waveguides coupled to an exterior region (waveguide, cavity, free space), this expansion of different Green's functions (vector potential, electric, magnetic) was extensively applied to determine the magnetic field due to the magnetic current source [8, 19, 25-31]. It appears that the computation of the Green's function in the form of a partial eigenfunction expansion was not that straightforward, particularly in the directional waveguide couplers or waveguides radiating in free space through longitudinal or tilted slots. For this class of problems there is a term in a double series expansion with zero indices (the (0,0) term) that corresponds to a contribution occurring in the source region only. It should also be noted that the (0,0) term does not represent a TE_{00} waveguide mode. It is obvious that this "mode" does not propagate in the waveguide, and this, in turn, resulted in a dilemma. The question was whether to exclude this term from the series expansion of the Green's function, or whether it must necessarily be included in the expansion to guarantee the complete representation of the Green's function in the source region. In some papers, this term was "hidden" by not showing the limits in the series expansion, but, for example, in [26], it was clearly stated that "the term with $(m,n)=(0,0)$ is omitted." In [32], it was demonstrated that the (0,0) "mode" is associated with the "power stored" inside the waveguide and should be included in the series. The discussion on this topic was continued in a series of comments [33-35] based on [27]. Finally, it became evident that the (0,0) term must be included in the Green's function series expansion.

The purpose of this paper is to attempt to clearly explain how the inclusion of the (0,0) term in the series expansion of the Green's function makes its representation complete in the source region. This is demonstrated by extracting the delta-function term directly from the series expansion, using principles of the theory of distributions. Moreover, it is shown that the delta-function term can be extracted only if the (0,0) term is included in the series, which is associated with completeness of eigenfunctions. Numerical results presented illustrate a comparative analysis for longitudinally coupled waveguides and for waveguides radiating in free space through longitudinal and tilted slots, emphasizing the importance of the (0,0) term to their frequency-dependent characteristics.

2. Problem Formulation and Electric Dyadic Green's Function

Consider an infinite rectangular waveguide with arbitrarily shaped apertures (slots) S_a^i placed in the perfectly conducting waveguide surface S_m, as shown in Figure 1. The waveguide interior is characterized by material parameters ε and μ (in general, ε and μ can include losses). The incident electric and magnetic fields are generated inside the waveguide by an impressed electric current source, $\mathbf{J}_{imp}(\mathbf{r})$, $\mathbf{r} \in V_{imp} \subset V$. The integral representation of the magnetic field inside the waveguide is given in a general form for arbitrarily shaped and arbitrarily oriented slots; however, in the analysis of the Green's dyadic, we will be particularly interested in the $\hat{\mathbf{z}}\hat{\mathbf{z}}$ component of the Green's function associated with the longitudinal slot. Also, it should be noted that the formulation presented below for the waveguide part of the slotted waveguide radiating in free space can be used in the analysis of waveguides coupled through the slots, or any waveguide-based aperture-coupled antennas, including microstrip and dielectric-resonator antennas.

The solution for the waveguide coupled to some exterior region is based on the integral-equation formulation for the equivalent magnetic surface current density, $\mathbf{K}_i$, induced on the surface of the apertures S_a^i. Thus, the total magnetic field inside the waveguide can be obtained as [18]

$$\mathbf{H}(\mathbf{r}) = \int_{V_{imp}} \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')\cdot\left[\nabla'\times\mathbf{J}_{imp}(\mathbf{r}')\right]dV' - j\omega\varepsilon\sum_i \int_{S_a^i} \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')\cdot\mathbf{K}_i(\mathbf{r}')dS', \quad (1)$$

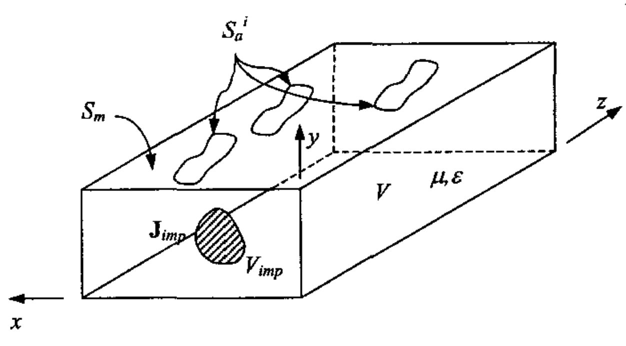

Figure 1. An infinite rectangular waveguide with arbitrarily shaped radiating slots.

where $\mathbf{K}_i(r) = -\hat{\mathbf{n}}_i \times \mathbf{E}_i(r)$, $\mathbf{r} \in S_a^i$; $\hat{\mathbf{n}}_i$ is the normal to the surface S_a^i pointing inwards to the waveguide; and the distribution $\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')$ is the electric dyadic Green's function of the second kind obtained for an infinite rectangular waveguide as the solution of the boundary-value problem

$$\nabla \times \nabla \times \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') - k^2 \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \bar{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}'), \quad \mathbf{r},\mathbf{r}' \in V,$$

$$\hat{\mathbf{n}} \times \nabla \times \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \bar{\mathbf{0}}, \quad \mathbf{r} \in \tilde{S}, \tag{2}$$

$$\hat{\mathbf{n}} \cdot \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \mathbf{0}, \ \mathbf{r} \in \tilde{S}.$$

In Equation (2), $\tilde{S} = S_m \cup \left(\cup_i S_a^i \right)$, $k = \omega\sqrt{\mu\varepsilon}$, $\hat{\mathbf{n}}$ is the inward normal to $\tilde{S}$, and we assume that $\bar{\mathbf{G}}_e$ is regular at infinity.

It should be noted that the volume integral in Equation (1) is ostensibly taken over the entire source region. The distribution $\bar{\mathbf{G}}_e$ will be separated into a principle-value term (associated with evaluating the integral of this term over a certain principle volume), and a source dyadic delta-function term (containing a multiplicative factor that is associated with the same principle volume). This decomposition is not unique, since it depends on the principle volume, although the sum of these two terms leads to the unique field.

The components of the Green's function are obtained in the form of an expansion over the complete system of eigenfunctions $\varphi_{mn}^{\alpha}(x,y)$ of a transverse Laplacian operator, $\nabla_{xy} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, with the one-dimensional characteristic Green's functions $f_{mn}^{\alpha\beta}(z,z')$, $\alpha,\beta = x,y,z$ in the waveguiding direction,

$$G_e^{\alpha\beta}(x,y,z;x',y',z') = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \varphi_{mn}^{\alpha}(x,y)\varphi_{mn}^{\alpha}(x',y') f_{mn}^{\alpha\beta}(z,z'). \tag{3}$$

Here, we are particularly interested in the $\hat{\mathbf{z}}\hat{\mathbf{z}}$ component of the Green's function, which is obtained in the form [18]

$$G_e^{zz}(\mathbf{r},\mathbf{r}') = \frac{1}{k^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\varepsilon_{0m}\varepsilon_{0n}}{ab}\left[k^2 + \gamma_{mn}^2 - 2\gamma_{mn}\delta(z-z')\right] \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\frac{e^{-\gamma_{mn}|z-z'|}}{2\gamma_{mn}}, \tag{4}$$

where $\gamma_{mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$, and ε_{0m} is the Neumann index such that $\varepsilon_{0m} = 1$ for $m = 0$ and $\varepsilon_{0m} = 2$ for $m \neq 0$.

It should be noted that the partial eigenfunction expansion, Equation (4), of the electric Green's function can be obtained as a direct solution of the problem in Equation (2), or by using a relationship between electric, $\bar{\mathbf{G}}_e$, and vector potential, $\bar{\mathbf{G}}_A$, Green's dyadics,

$$\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right)\cdot\bar{\mathbf{G}}_A(\mathbf{r},\mathbf{r}'). \tag{5}$$

For the $\hat{\mathbf{z}}\hat{\mathbf{z}}$ component, it reduces to

$$\bar{\mathbf{G}}_e^{zz}(\mathbf{r},\mathbf{r}') = \left(1 + \frac{1}{k^2}\frac{\partial^2}{\partial z^2}\right)\mathbf{G}_A^{zz}(\mathbf{r},\mathbf{r}'), \tag{6}$$

where $\mathbf{G}_A^{zz}(\mathbf{r},\mathbf{r}')$ is obtained as [18]

$$\mathbf{G}_A^{zz}(\mathbf{r},\mathbf{r}') = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{0m}\varepsilon_{0n}}{ab}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right) \cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\frac{e^{-\gamma_{mn}|z-z'|}}{2\gamma_{mn}} \tag{7}$$

Derivatives in Equation (6) can be applied term-by-term in the sense of distributions. Indeed, the resulting series for the electric Green's dyadic is divergent in the classical sense at $z = z'$, but the integral in Equation (1) of the Green's function with sufficiently smooth currents is well defined.

Also, it should be noted that the first term, $(m,n) = (0,0)$, of all components of the electric, $G_e^{\alpha\beta}$, and vector-potential, $G_A^{\alpha\alpha}$ (diagonal tensor), Green's dyadics vanishes, except for the $\hat{\mathbf{z}}\hat{\mathbf{z}}$ component. Although this combination of m and n corresponds to a value of γ_{mn} that is equal to that of the unbounded medium $(\gamma_{00} = -jk)$, this term still represents a solution to the eigenvalue problem for the differential operator ∇_{xy}; it satisfies the boundary conditions and, therefore, should not be discarded on the basis that it does not represent a waveguide mode.

In fact, the exclusion of the (0,0) term from the series expansion in Equation (4) violates the property of completeness of the cross-sectional eigenfunctions. This can be shown by separating the series, understood in the sense of distributions, into two parts as follows:

$$\begin{aligned}\mathbf{G}_e^{zz}(\mathbf{r},\mathbf{r}') = &-\frac{1}{k^2}\delta(z-z')\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{0m}\varepsilon_{0n}}{ab} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\\ &+\frac{1}{k^2}PV\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{0m}\varepsilon_{0n}}{ab}\left(k^2+\gamma_{mn}^2\right) \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\frac{e^{-\gamma_{mn}|z-z'|}}{2\gamma_{mn}},\end{aligned} \tag{8}$$

where PV indicates a slice principle-volume integration [13] with normal to the z-axis, which naturally arises from the one-dimensional Green's function $f_{mn}^{zz}(z,z')$ in Equation (3), and where in the first term in Equation (8) we have applied the distributional property [36]

$$\delta(z-z')e^{-\gamma_{mn}|z-z'|} = \delta(z-z'). \tag{9}$$

(In general, $\delta(z-z')f(z-z')=\delta(z-z')f(z=z')$ for f continuous at $z=z'$). Note that the decomposition in Equation (8), and later in Equations (11) and (12), is equivalent to the procedure described in [37, Sections 3.14 and 3.26]. The well-known spectral expansion of the two-dimensional delta function (valid in $[o,a]\times[0,b]$) can be obtained as [17, 18]

$$\delta(x-x')\delta(y-y')=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{0m}\varepsilon_{0n}}{ab}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right). \quad (10)$$

Of course, the (0,0) term must necessarily be included in the expansion of Equation (10) for the equality to hold in the distributional sense. Now, taking into account the spectral representation of Equation (10), the expression for the $\hat{z}\hat{z}$ component of the Green's function in Equation (8) can be written as

$$G_e^{zz}(\mathbf{r},\mathbf{r}')=-\frac{1}{k^2}\delta(\mathbf{r}-\mathbf{r}')+\frac{1}{k^2}PV\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{0m}\varepsilon_{0n}}{ab}\left(k^2+\gamma_{mn}^2\right)\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\frac{e^{-\gamma_{mn}|z-z'|}}{2\gamma_{mn}}. \quad (11)$$

It is worth noting that the first term in the series expansion, $(m,n)=(0,0)$, vanishes due to $\gamma_{00}^2=-k^2$. The remaining terms correspond to the TE and TM propagating and evanescent modes in the waveguide. The expression of Equation (11) for the Green's function explicitly shows that the delta-function term is extracted from the partial eigenfunction expansion, which makes it complete in the source region. It can be written in a similar form introduced in [4] for the electric Green's dyadic due to the electric current

$$\overline{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')=PV\left\{\overline{\mathbf{G}}_e^0(\mathbf{r},\mathbf{r}')\right\}-\frac{1}{k^2}\hat{z}\hat{z}\delta(\mathbf{r}-\mathbf{r}'). \quad (12)$$

In the case of a longitudinal slot, the $\hat{z}\hat{z}$ component of $\overline{\mathbf{G}}_e^0(\mathbf{r},\mathbf{r}')$ can be written in the form

$$G_e^{0zz}(\mathbf{r},\mathbf{r}')=\frac{1}{k^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{0m}\varepsilon_{0n}}{ab}\left(k^2+\gamma_{mn}^2\right)\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\frac{e^{-\gamma_{mn}|z-z'|}}{2\gamma_{mn}}. \quad (13)$$

It should be noted that the $\overline{\mathbf{G}}_e^0(\mathbf{r},\mathbf{r}')$ part (13) of the Green's dyadic, Equation (12), results in a divergent series when $z=z'$, even though $x\neq x'$ and/or $y\neq y'$. This is associated with the slice-pillbox exclusion volume of the source plane at z'.

While mathematically the existence of the (0,0) term is necessary to satisfy completeness in the source region, this term can also be regarded from a different perspective. Equation (4) may be rewritten in the form

$$G_e^{zz}(\mathbf{r},\mathbf{r}')=-\frac{1}{abk^2}\delta(z-z')+\frac{1}{k^2}PV\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_{mn}}{ab}\left[k^2+\gamma_{mn}^2-2\gamma_{mn}\delta(z-z')\right]\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\frac{e^{-\gamma_{mn}|z-z'|}}{2\gamma_{mn}}, \quad (14)$$

where

$$\varepsilon_{mn}=\begin{cases}0, & m=n=0,\\ 2, & m=0 \text{ or } n=0,\ m\neq n,\\ 4, & m,n\neq 0.\end{cases}$$

The expression in Equation (14) implies that the (0,0) "mode" exists in the source plane $z-z'$, but is not allowed to propagate in the waveguide (vanishes at any point $z\neq z'$). This "mode" is associated with the "power stored" in the vicinity of the source plane [32]. Note also that it has no variation with respect to x or y variables. Still, this does not violate the boundary conditions, as the longitudinal magnetic field should have the maximum value on the perfectly conducting walls of the waveguide, including the special case of a constant value all over the cross section. It is worth mentioning that this term will also occur in the dual problem: the electric field produced by a longitudinal electric current source in a waveguide with perfectly magnetic walls.

In the class of problems where a longitudinal slot couples the waveguide to an arbitrary exterior region (another waveguide, free space, etc.), this term has a significant effect on the scattering parameters. Regarding the application of tilted slots, where the longitudinal component of the magnetic current is associated with the $\hat{z}\hat{z}$ component of the electric dyadic Green's function discussed above, this effect decreases as the slot tilts from the longitudinal direction, until it vanishes for a transverse slot.

It was noticed, however, that some previous publications discard this term explicitly [26] or implicitly [28]. The effect of discarding this term will be illustrated in the next section.

3. Numerical Results and Discussion

The analysis of the Green's function presented in the previous section has been validated numerically and compared to the numerical and experimental results published in the literature for a few representative structures, including directionally (in the waveguiding direction) coupled rectangular waveguides, and waveguides radiating in free space through longitudinal and tilted slots. In all examples, a standard hollow waveguide, of dimensions 2.286 cm × 1.016 cm has been used to operate in the X band.

In the first example of two longitudinally coupled waveguides, the results of the scattering parameters obtained using a Method of Moments (MoM) numerical code, with the electric dyadic Green's function discussed in the previous section, and a Finite-Difference Time-Domain (FDTD) commercial software package [38], are compared with the results published in [28]. Figures 2 and 3 show the coupling coefficient (S_{31}) of a broad-wall longitudinal slot coupler between two identical rectangular waveguides with the common wall being of 0 mm and 2 mm thickness, respectively. In both cases, the slot dimensions were 1.6 cm × 1 mm, and its centerline was at 0.943 cm from the narrow wall. The results obtained by the MoM technique without the (0,0) term in the Green's function expansion show good agreement with the results presented in [28, Fig. 3], leading to the conclusion that this term was missing in the calculation of the S parameters in [28]. The other two MoM curves in Figures 2 and 3 were obtained with the (0,0) term in the expansion, and show good agreement with the results generated by the commercial FDTD software [38]. Figure 4 shows the results for the same structure, but with a slot of dimensions 1.56 cm × 1 mm and a wall thickness of 1.27 mm. Again, as in the previous figures, the results obtained by the MoM technique with and without the (0,0) term are compared to those published in [28, Fig. 5] and to the FDTD results. It can be seen that the exclusion of the (0,0) term results in erroneous values of the scattering parameters. For this reason, we conclude that the results presented in [28] (including the experimental data) are incorrect.

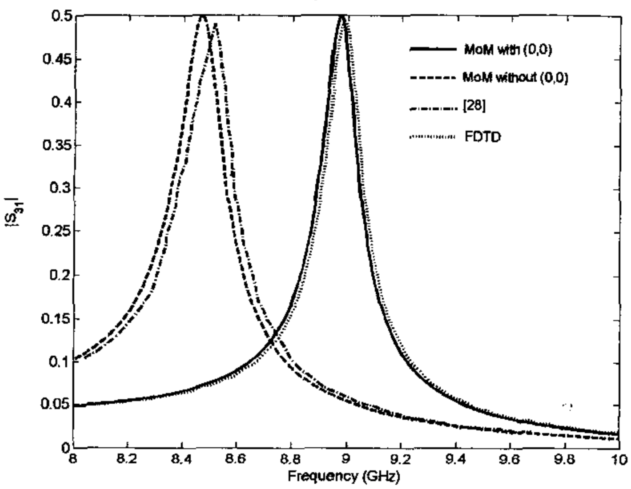

Figure 2. The coupling coefficient (S_{31}) as a function of frequency for a longitudinal-slot waveguide coupler with the following dimensions: the waveguide was 2.286 cm × 1.016 cm, the slot was 1.6 cm × 1 mm, the slot centerline was at 0.943 cm from the narrow wall, and the wall thickness was 0 mm.

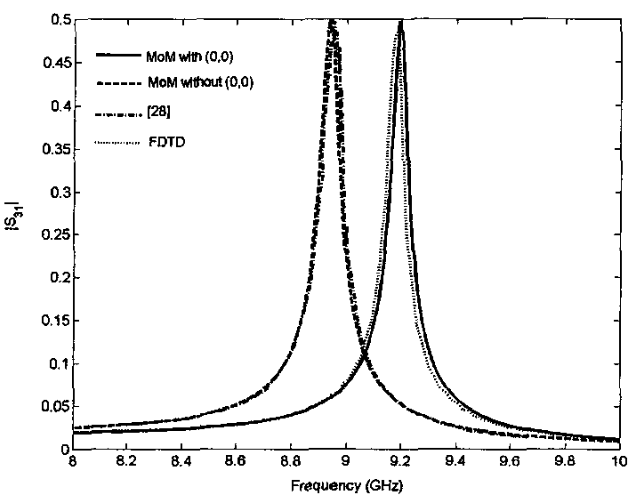

Figure 3. The coupling coefficient (S_{31}) as a function of frequency for a longitudinal-slot waveguide coupler with a wall thickness of 2 mm.

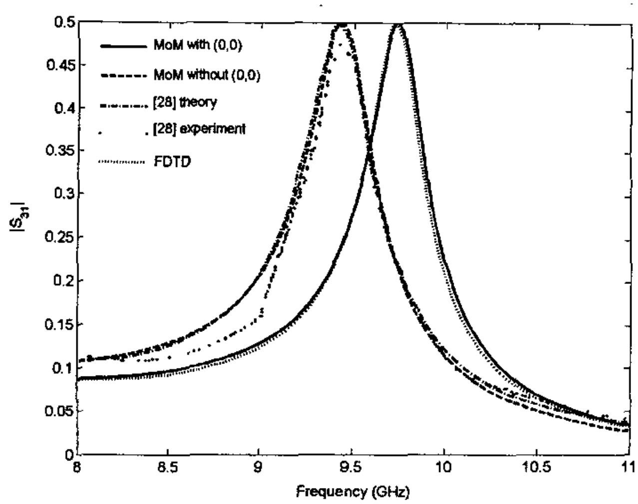

Figure 4. The coupling coefficient (S_{31}) as a function of frequency for a longitudinal-slot waveguide coupler with the following dimensions: the waveguide was 2.286 cm × 1.016 cm, the slot was 1.56 cm × 1 mm, the slot centerline was at 0.643 cm from the narrow wall, and the wall thickness was 1.27 mm.

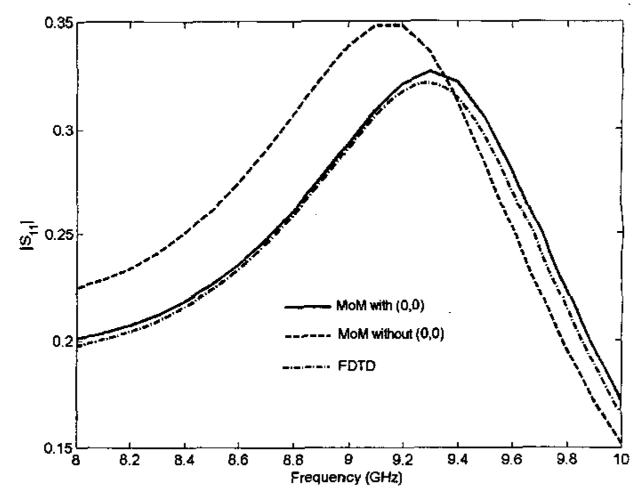

Figure 5. The return loss (S_{11}) as a function of frequency for a longitudinal-slot waveguide radiator with the following dimensions: the waveguide was 2.286 cm × 1.016 cm, the slot was 1.6 cm × 1 mm, the slot centerline was at 0.343 cm from the narrow wall, and the wall thickness was 1.27 mm.

In the second example, the waveguide was radiating in free space through the longitudinal slot, and the results obtained by the MoM technique are compared to those generated by the commercial FDTD software [38]. Figure 5 exhibits the effects of the (0,0) term on the scattering characteristics of the radiating waveguide.

Table 1. A comparison of theory and experiment ($|S_{11}|$).

Tilt (°)	Theory [30]	Experiment [30]	Theory with (0,0) term	Theory without (0,0) term
5	$0.161\angle 172°$	$0.157\angle 167°$	$0.159\angle 176°$	$0.178\angle 176°$
10	$0.172\angle 153°$	$0.168\angle 151°$	$0.170\angle 156°$	$0.189\angle 158°$
20	$0.208\angle 121°$	$0.203\angle 116°$	$0.205\angle 123°$	$0.222\angle 127°$

In the third example, the results were obtained for the waveguide radiating in free space through a tilted slot. Table 1 compares the results obtained from the present theory (with and without the (0,0) term) for the return loss of a tilted slot radiator at resonance on the broad wall of an infinite waveguide with those obtained experimentally and theoretically in [30]. The waveguide dimensions were the same as in the previous examples, the slot dimensions were 1.6 cm × 1.5875 mm, the wall thickness was 1.27 mm, and the slot center was at 0.381 cm from the waveguide centerline. The MoM numerical solution was based on the rooftop expansion and testing with nine unknowns and used 50 × 50 terms in the Green's function expansion. It should be noted that the resonance occurs at 9 GHz if the (0,0) term is included in the Green's function expansion, and at 8.8 GHz if it is not.

4. Conclusion

It was shown here that based on the theory of distributions, the delta-function term can be extracted from the partial eigenfunction expansion of the electric Green's dyadic for a rectangular waveguide. It was also shown that the first term (the (0,0) term) of the double series expansion, which does not correspond to any of the waveguide modes, has to be included in the calculation of the series to guarantee completeness of the eigenfunctions in the source region. The effect of this term on the scattering characteristics of directionally coupled waveguides and waveguides radiating in free space through longitudinal and tilted slots was illustrated, where the results obtained using the MoM and FDTD techniques were compared to those published in the literature with and without this term.

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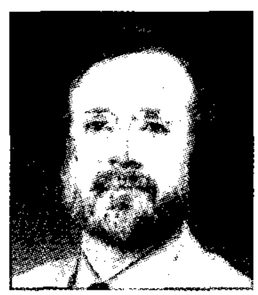

Allen W. Glisson received the BS, MS, and PhD degrees in Electrical Engineering from the University of Mississippi in 1973, 1975, and 1978, respectively. In 1978, he joined the faculty of the University of Mississippi, where he is currently a Professor and Chair of the Department of Electrical Engineering. He was selected as the Outstanding Engineering Faculty Member in 1986, and again in 1996. He received a Ralph R. Teetor Educational Award in 1989, and in 2002 he received the Faculty Service Award in the School of Engineering.

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Editor's Comments *Continued from page 8*

ties for inadequately identified multiple submission range from rejection of the paper and a formal warning to the author being banned from publishing in IEEE publications for periods of years.

The key to all of this is very simple: if you use your own prior work or someone else's work or ideas, provide full credit and citation!

Our Feature Articles

A layer of charge or current at the boundary of a material is usually associated with a discontinuity in the electromagnetic field at the boundary. Similarly, a discontinuity in the field is often assumed to have certain implications regarding surface charges and/or currents at a material boundary. As Jack Nachamkin shows in his feature article, the latter may or may not be true. He shows that an accurate calculation of surface currents and charge layers at a material boundary must include not only the electromagnetic fields, but the stresses (and possibly the strains) and the transfer of momentum at the boundary. He presents a correct method for calculating the surface currents and charges at a boundary, based on momentum transfer. This more-complete picture of the interaction among fields, surface currents and charges, and momentum at the boundary of a material has significant implications for a variety of interesting problems. In particular, it provides a basis for demonstrating that spatially bounded spherical plasma regions are consistent with the physics and mathematics governing electromagnetic fields and plasmas. There is also basis for speculating that such plasma regions could have a degree of stability. This could be a possible explanation for some forms of ball lightning. This is a very thought-provoking article, with a variety of other potential applications, as well. I urge you to read it carefully.

Raj Mittra has provided us with an interesting overview of part of the field of computational electromagnetics. The emphasis is on problems that challenge today's computational methods and resources, focusing on problems that have been worked on at the EMC Laboratory at Penn State. The problems considered include large planar arrays, a variety of problems involving frequency-selective surfaces, conformal arrays, EMC/EMI problems on complex platforms, coupling between aperture antennas in situations where ray techniques cannot be used, and EMI for systems located inside buildings. A description is given of several approaches taken to addressing such problems, with particular emphasis on the Characteristic Basis Function Method and on the Windowed Plane Wave Spectrum approach, developed at Penn State. As stated in the conclusion, it is hoped that other methods will be (and, perhaps, already have been) applied to some of the problems described in this article. I will add that where such results are or become available, here is an invitation to share descriptions of them with the readers of this *Magazine.*

There has been a substantial amount published on the complete representation of the electric dyadic Green's function for cylindrical and rectangular waveguides. There has also been some controversy related to this, particularly in connection with whether or not – and in what manner – the first, or "(0,0)," term in the dou-

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