On the epsilon near zero condition for spatially dispersive materials

2013 New J. Phys. 15 123027

(http://iopscience.iop.org/1367-2630/15/12/123027)

New Journal of Physics
The open access journal for physics

On the epsilon near zero condition for spatially dispersive materials

Ebrahim Forati and George W Hanson[1]
Electrical Engineering Department, University of Wisconsin-Milwaukee, Milwaukee, WI 53211, USA
E-mail: george@uwm.edu

New Journal of Physics **15** (2013) 123027 (11pp)
Received 18 September 2013, revised 17 November 2013
Accepted for publication 26 November 2013
Published 17 December 2013
Online at http://www.njp.org/
doi:10.1088/1367-2630/15/12/123027

Abstract. The epsilon-near-zero (ENZ) condition in natural and artificial plasmas is considered for spatially dispersive materials. In the presence of spatial dispersion the ENZ condition must be judged by vanishing of the electric displacement field in real-space. Unlike the simple case of local materials where ENZ occurs at the plasma frequency, in spatially dispersive materials the matter is more complicated. To consider the spatially dispersive case, we obtain the momentum-dependent permittivity in real-space, and define a characteristic length parameter, in addition to the Debye length, which governs polarization screening. Using this formulation, conditions are investigated under which the electric displacement field (equivalently, the real-space permittivity) can vanish or be strongly diminished, even in the presence of spatial dispersion, implementing an ENZ material.

S Online supplementary data available from stacks.iop.org/NJP/15/123027/mmedia

[1] Author to whom any correspondence should be addressed.

Contents

1. Introduction	2
2. Permittivity tensor and polarization vector in space domain	4
3. Discussion of the results and the establishment of the epsilon-near-zero condition	5
4. Conclusion	9
Appendix	9
References	11

1. Introduction

Materials with effective permittivity of approximately zero, also known as epsilon-near-zero (ENZ) materials [1], have become an important topic of research with a variety of applications. For example, enhancing the radiation directivity of antennas [2], supercoupling [3, 4], transforming curved wavefronts into planar ones [5], implementing optical nano circuit concepts [6], cloaking [7, 8], designing lenses with enhanced focusing [9] and tailoring the radiation pattern of antennas [10] are recent applications of ENZ materials. However, to our knowledge, in previous work on ENZ applications spatial dispersion of the material was ignored in establishing the ENZ condition, which, for local materials, occurs exactly at the plasma frequency. This is often a reasonable approximation for natural materials (NM) where spatial dispersion is fairly weak (e.g. typical semiconductors (SCs), metals or high-density plasmas). However, this may not be a good approximation for artificial materials with strong spatial dispersion, such as wire media [11–15].

The aim of this work is to address the following question: under what conditions can we achieve an ENZ medium (for which $\mathbf{D} \simeq \mathbf{0}$) in the presence of spatial dispersion. To answer this question, we provide the 'space-domain' form of the permittivity $\underline{\varepsilon}(\mathbf{r})$ appropriate for forming the space-domain relation between displacement and electric field, $\mathbf{D}(\mathbf{r}) = \int \underline{\varepsilon}\,(\mathbf{r}-\mathbf{r}') \cdot \mathbf{E}\,(\mathbf{r}')\, \mathrm{d}^3\mathbf{r}'$. Using this, we introduce a screening parameter k_α that relates polarization to the total electric field, in a similar way that k_{D}, the Debye wavenumber, relates polarization to the incident electric field.

We consider the momentum-dependent permittivity tensor in the spatial–temporal Fourier transform domain as

$$\frac{\underline{\varepsilon}(\mathbf{q})}{\varepsilon_0} = \varepsilon_{\mathrm{h}}\underline{\mathbf{I}} - \kappa\left(\underline{\mathbf{I}} - \frac{1}{q^2-\alpha^2}\mathbf{q}\mathbf{q}\right). \tag{1}$$

This general form models a wide range of NM, including many SCs, plasmas and metals [16], as well as the isotropic connected wire medium (ICWM), depicted in figure 1, which acts as an artificial plasma [15, 17, 18]. In (1), $\underline{\mathbf{I}}$ is the identity tensor, ε_{h} is the relative permittivity (for a wire medium this is the permittivity of the host medium), ε_0 is the permittivity of vacuum,

$$\alpha^2 = -\frac{\mathrm{j}\omega}{D}, \quad \kappa = \frac{\mathrm{j}\sigma}{\omega\varepsilon_0}, \tag{2}$$

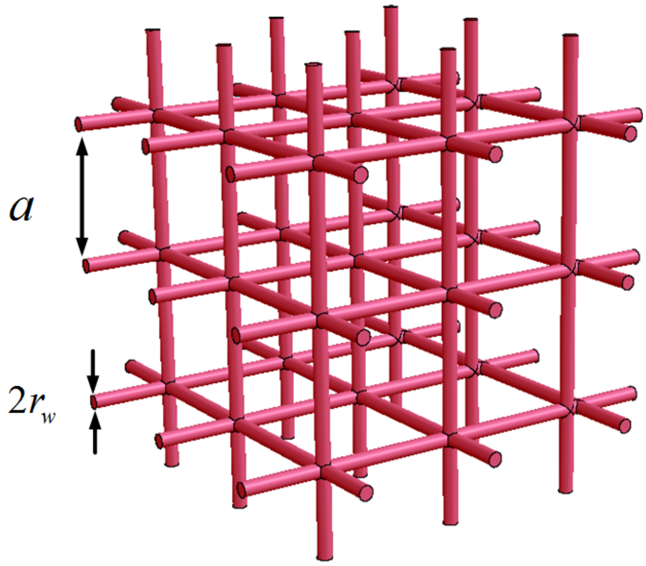

Figure 1. ICWM with wire period (a) and radius (r_w).

where σ is the conductivity ($\mathrm{S\,m^{-1}}$), D is the diffusion coefficient ($\mathrm{m^2\,s^{-1}}$), ω is the radian frequency (time dependence is $e^{j\omega t}$) and $\mathbf{q}$ is the spatial Fourier transform wavenumber $\mathbf{q} \leftrightarrow \mathbf{r}$

$$\mathcal{F}\{f(\mathbf{r})\} = F(\mathbf{q}) = \int f(\mathbf{r})\mathrm{e}^{-\mathrm{j}\mathbf{q}\cdot\mathbf{r}}\,\mathrm{d}^3\mathbf{r}, \tag{3}$$

$$f(\mathbf{r}) = \frac{1}{(2\pi)^3}\int F(\mathbf{q})\mathrm{e}^{\mathrm{j}\mathbf{q}\cdot\mathbf{r}}\,\mathrm{d}^3\mathbf{q}, \tag{4}$$

where $\mathbf{r}$ is the position vector.

We assume all material losses are incorporated in the conductivity. For NM we assume conductivity in the usual Drude form

$$\sigma = \frac{\omega_\mathrm{p}^2\varepsilon_0}{(j\omega + \tau^{-1})}, \tag{5}$$

where $\omega_\mathrm{p}^2 = Nq_\mathrm{e}^2/\varepsilon_0 m_\mathrm{e}$ (ω_p is the plasma frequency and N is the number density) and τ is the relaxation time—typical values for SCs and metals are $\tau \sim 10$–$100\,\mathrm{fs}$. For the isotropic wire medium the effective conductivity can be expressed in the same Drude form [18] upon defining $\tau^{\mathrm{WM}} \equiv f_\mathrm{v}\sigma_m/\omega_\mathrm{p}^2\varepsilon_0$, where σ_m is the assumed real-valued conductivity of the wires (the permittivity of the wires is $\varepsilon_m = 1 - \mathrm{j}\sigma_m/\omega\varepsilon_0$), ω_p is the plasma frequency $(\omega_\mathrm{p}a)^2 \cong 2\pi c^2/\ln(a^2/4r_\mathrm{w}/(a-r_\mathrm{w}))$ where a and r_w are wire period and radius, respectively and f_v is the volume fraction of wires, $f_\mathrm{v} = \pi r_\mathrm{w}^2/a^2$. Typical values of τ^{WM} are on the order of μs, although $(\tau^{\mathrm{WM}})^{-1} \to 0$ for perfect electrical conducting (PEC) wires in a lossless host medium.

The diffusion coefficient in (2) is

$$D = \frac{\beta}{\left(\mathrm{j}\omega + \tau^{-1}\right)}. \tag{6}$$

For NM $\beta^{\mathrm{NM}} = \langle v^2\rangle/3$, with $\langle v^2\rangle$ being the electron mean-square velocity [19]. For plasmas and SCs $\langle v^2_{\mathrm{thermal}}\rangle = 3k_{\mathrm{B}}T/m$, but for good metals $\langle v^2\rangle = 3v_{\mathrm{F}}^2/5$, where v_{F} is the electron Fermi velocity. For isotropic wire media $D^{\mathrm{ICWM}} = c^2\sigma^{\mathrm{ICWM}}/l_0\varepsilon_0\varepsilon_{\mathrm{h}}\omega_{\mathrm{p}}^2$ [18], such that

$$\beta^{\mathrm{WM}} = \frac{c^2}{l_0\varepsilon_{\mathrm{h}}} = \frac{v_{\mathrm{h}}^2}{l_0}, \tag{7}$$

where $v_{\mathrm{h}} = 1/\sqrt{\mu_0\varepsilon_0\varepsilon_{\mathrm{h}}}$ is the electromagnetic phase velocity in the nondispersive host and l_0 is a weak function of wire period with typical values of $l_0 \sim 2$–3 [20]; note the similarity between $\beta^{\mathrm{WM}} = v_{\mathrm{h}}^2/l_0 \simeq v_{\mathrm{h}}^2/3$ and $\beta^{\mathrm{NM}} = \langle v^2\rangle/3$. For lossless NM or PEC wire materials with a nonabsorbing host, $\kappa = \omega_{\mathrm{p}}^2/\omega^2$, $\alpha^2 = \omega^2/\beta$.

It is evident from (1) that in the local limit ($q \to 0$) and assuming lossless NM or wire media with perfectly conducting wires ($\varepsilon_m \to -\infty$), the space-domain permittivity will be

$$\frac{\underline{\varepsilon}}{\varepsilon_0} = \varepsilon_{\mathrm{h}}\left(1 - \frac{\omega_{\mathrm{p}}^2}{\varepsilon_{\mathrm{h}}\omega^2}\right)\underline{\mathbf{I}}, \tag{8}$$

which is the equation commonly used in the design of ENZ materials. The permittivity (8) becomes zero at the frequency $\omega = \omega_{\mathrm{p}}/\sqrt{\varepsilon_{\mathrm{h}}}$.

In the following, we find the inverse Fourier transform of (1), and the polarization (and therefore the electric displacement field) in terms of the total field inside the material. In the appendix we include for completeness the relation between polarization and the scattered and incident fields, which leads to the idea of a Debye length. From the polarization in the space domain we will see that the Debye length is not the best parameter for considering the ENZ condition. A new wavenumber leading to a different characteristic screening length will be introduced to study the electric displacement inside a spatially dispersive material, and conditions will be investigated under which the electric displacement can vanish.

2. Permittivity tensor and polarization vector in space domain

The inverse Fourier transform of (1) is easily seen to be

$$\frac{\underline{\varepsilon}(\mathbf{r})}{\varepsilon_0} = (\varepsilon_{\mathrm{h}} - \kappa)\,\delta(\mathbf{r})\underline{\mathbf{I}} - \kappa\nabla\nabla\frac{\mathrm{e}^{-\mathrm{j}\alpha r}}{4\pi r}, \tag{9}$$

where $\delta(\mathbf{r})$ is the Dirac delta function and we have used the Fourier transform identities

$$\mathcal{F}\left\{\frac{\mathrm{e}^{-\mathrm{j}\alpha r}}{4\pi r}\right\} = \frac{1}{q^2 - \alpha^2}, \tag{10}$$

$$\mathcal{F}\{\nabla\nabla g(\mathbf{r})\} = -\mathbf{q}\mathbf{q}\mathcal{F}\{g(\mathbf{r})\}. \tag{11}$$

The derivatives are easily carried out to yield

$$\frac{\underline{\varepsilon}(\mathbf{r})}{\varepsilon_0} = (\varepsilon_{\mathrm{h}} - \kappa)\,\delta(\mathbf{r})\underline{\mathbf{I}} - \kappa\left\{(3\hat{\mathbf{r}}\hat{\mathbf{r}} - \underline{\mathbf{I}})\left(\frac{1}{r^2} - \frac{\alpha}{\mathrm{i}r}\right) - \alpha^2\hat{\mathbf{r}}\hat{\mathbf{r}}\right\}\frac{\mathrm{e}^{-\mathrm{j}\alpha r}}{4\pi r}. \tag{12}$$

Equation (9) is the permittivity tensor in the 'space domain' (it is not actually in real space because of the delta function—it is a quantity to be integrated) which relates the electric displacement vector $\mathbf{D}(\mathbf{r})$ and the electric field vector $\mathbf{E}(\mathbf{r})$ as $\mathbf{D}(\mathbf{r}) = \underline{\varepsilon}(\mathbf{r}) * \mathbf{E}(\mathbf{r})$, where * denotes convolution,

$$\mathbf{D}(\mathbf{r}) = \int \underline{\varepsilon}(\mathbf{r}-\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')\, \mathrm{d}^3\mathbf{r}'. \tag{13}$$

In order to determine the displacement field in the space domain, we need a relation between polarization and total field. In the following it is useful to introduce the screening wavenumber $k_\alpha \equiv \mathrm{j}\alpha = -\sqrt{\mathrm{j}\omega/D}$. Using (1) in the definition of the polarization,

$$\begin{aligned}\mathbf{P}(\mathbf{q}) &= \mathbf{D}(\mathbf{q}) - \varepsilon_0\mathbf{E}(\mathbf{q}) = \left(\underline{\varepsilon}(\mathbf{q}) - \varepsilon_0\underline{\mathbf{I}}\right) \cdot \mathbf{E}(\mathbf{q}) \\ &= \varepsilon_0 \left((\varepsilon_\mathrm{h} - 1 - \kappa)\, \underline{\mathbf{I}} + \kappa \frac{\mathbf{q}\mathbf{q}}{\left(q^2 + k_\alpha^2\right)} \right) \cdot \mathbf{E}(\mathbf{q}),\end{aligned} \tag{14}$$

where $\mathbf{E}$ is the total electric field inside the material. Decomposing (14) into parallel and perpendicular components with respect to $\mathbf{q}$,

$$\frac{\mathbf{P}_\parallel(\mathbf{q})}{\varepsilon_0} = (\varepsilon_\mathrm{h} - 1 - \kappa)\, \mathbf{E}_\parallel(\mathbf{q}) + \kappa \frac{q^2\mathbf{E}_\parallel(\mathbf{q})}{\left(q^2 + k_\alpha^2\right)}, \tag{15}$$

$$\frac{\mathbf{P}_\perp(\mathbf{q})}{\varepsilon_0} = (\varepsilon_\mathrm{h} - 1 - \kappa)\, \mathbf{E}_\perp(\mathbf{q}) \tag{16}$$

in the transform domain, and

$$\frac{\mathbf{P}_\parallel(\mathbf{r})}{\varepsilon_0} = (\varepsilon_\mathrm{h} - 1 - \kappa)\, \mathbf{E}_\parallel(\mathbf{r}) - \kappa \left(\nabla^2\mathbf{E}_\parallel(\mathbf{r}) * \frac{\mathrm{e}^{-k_\alpha r}}{4\pi r} \right), \tag{17}$$

$$\frac{\mathbf{P}_\perp(\mathbf{r})}{\varepsilon_0} = (\varepsilon_\mathrm{h} - 1 - \kappa)\, \mathbf{E}_\perp(\mathbf{r}) \tag{18}$$

in the space domain. These show that the longitudinal polarization has a term that is local with the total longitudinal field, as well as a nonlocal contribution that decays according to the screening length $L_\alpha = 2\pi/Re\,(k_\alpha)$ (or, for imaginary-valued wavenumber the nonlocal contribution decays algebraically). In the absence of spatial dispersion ($D = 0$), $L_\alpha \to 0$ and the relation between polarization and total field is local.

3. Discussion of the results and the establishment of the epsilon-near-zero condition

In the presence of spatial dispersion the ENZ condition is $\mathbf{D}(\mathbf{r}) \simeq 0$. From (17) and (18),

$$\mathbf{D}_\parallel(\mathbf{r}) = \varepsilon_0\, (\varepsilon_\mathrm{h} - \kappa)\, \mathbf{E}_\parallel(\mathbf{r}) - \varepsilon_0\kappa\nabla^2\mathbf{E}_\parallel(\mathbf{r}) * \frac{\mathrm{e}^{-k_\alpha r}}{4\pi r}, \tag{19}$$

$$\mathbf{D}_\perp(\mathbf{r}) = \varepsilon_0\, (\varepsilon_\mathrm{h} - \kappa)\, \mathbf{E}_\perp(\mathbf{r}). \tag{20}$$

Now, we can consider three different scenarios:

(i) *The electric field is transverse (i.e. perpendicular to* $\mathbf{q}$*):* in this case we only need consider (20), and the ENZ condition can be satisfied simply by setting $\varepsilon_\mathrm{h} = \kappa$, which is equivalent to $\varepsilon^\mathrm{T} = 0$, which is the usual ENZ condition for a local material. As an example, if the electric field is associated with a transverse electric (TE) traveling wave, the permittivity becomes exactly zero (or equivalently, $\mathbf{D} = \mathbf{0}$) at the plasma frequency.

(ii) *The electric field is longitudinal (i.e. parallel to* $\mathbf{q}$*):* in this case we only need consider (19), and the ENZ condition is still possible by setting (19) to be zero, leading to

$$\varepsilon^{\mathrm{L}}(\mathbf{r}) * \mathbf{E}(\mathbf{r}) = 0, \tag{21}$$

where

$$\varepsilon^{\mathrm{L}}(\mathbf{r}) = (\varepsilon_{\mathrm{h}} - \kappa)\,\delta(\mathbf{r}) - \kappa\nabla^2\left(\frac{\mathrm{e}^{-k_\alpha r}}{4\pi r}\right). \tag{22}$$

Unlike ε^{T}, the longitudinal permittivity ε^{L} must be convolved with the electric field, and (21) does not simply lead to $\varepsilon^{\mathrm{L}} = 0$; the condition depends on $\mathbf{E}(\mathbf{r})$ in general. As an example, for a purely longitudinal field having the form $\mathbf{E}(\mathbf{r}) = \mathrm{e}^{-\mathrm{j}\mathbf{k}.\mathbf{r}}\hat{\mathbf{k}}$, then

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0\mathbf{E}(\mathbf{r})\left(\varepsilon_{\mathrm{h}} + \frac{\kappa\alpha^2}{k^2 - \alpha^2}\right), \tag{23}$$

for which the ENZ condition is $\varepsilon_{\mathrm{h}} = \kappa\left(1 - k^2/\alpha^2\right)^{-1}$. In the absence of spatial dispersion ($D = 0$), $\alpha^2 \to \infty$ and, as expected, the ENZ condition for longitudinal fields becomes the same as for transverse fields,

$$\varepsilon_{\mathrm{h}} = \kappa. \tag{24}$$

For lossless wire media (24) becomes

$$\omega = \frac{\omega_{\mathrm{p}}}{\sqrt{\epsilon_{\mathrm{h}}\left(1 - \frac{1}{l_0}\right)}}, \tag{25}$$

so that the effective plasma frequency is larger in the event of spatial dispersion then if spatial dispersion were absent (spatial dispersion can be set to zero by setting $l_0 = 0$ which leads to $D = 0$ and $\omega = \omega_{\mathrm{p}}/\sqrt{\epsilon_{\mathrm{h}}}$). It should be noted that a purely longitudinal field generally does not exist (an exception is a local material exactly at the plasma frequency), but here we consider this case since the next scenario is a combination of the first two cases.

(iii) *The electric field is neither purely transverse nor purely longitudinal (i.e. neither parallel nor perpendicular to* $\mathbf{q}$*):* in this case, the ENZ condition can be obtained only if

$$\varepsilon^{\mathrm{T}} = 0 \tag{26}$$

and (21) simultaneously, which leads to, from (19) and (20),

$$\nabla^2\mathbf{E}_{\parallel}(\mathbf{r}) * \frac{\mathrm{e}^{-k_\alpha r}}{4\pi r} = 0. \tag{27}$$

Equation (27) cannot be satisfied in general, and it depends on the electric field. However, if k_α has a large real part, (27) can be approximately satisfied independent of the electric field due to strong screening.

In summary, if we have a purely transverse field (such as a TE wave), the ENZ condition is satisfied as occurs for local materials. For purely longitudinal fields the ENZ condition can also be obtained, (24), although at a different frequency from the local case. And, for the general case the ENZ condition cannot be exactly obtained, although if sufficient field screening occurs via k_α the ENZ condition can be approximately satisfied.

Table 1. Normalized characteristic screening and Debye length of some sample materials.

Material	ω/ω_p	L_D/λ_0	L_α/λ_0
Gold	0.8	2.78×10^{-3}	1.79
Gold	1.2	2.69	1.48
SC ($N=10^{20}\,\mathrm{m^{-3}}$)	0.8	8.35×10^{-5}	8.36×10^{-5}
SC ($N=10^{20}\,\mathrm{m^{-3}}$)	1.2	1.02×10^{-4}	1.02×10^{-4}
SC ($N=10^{24}\,\mathrm{m^{-3}}$)	0.8	1.52×10^{-3}	1.61×10^{-3}
SC ($N=10^{24}\,\mathrm{m^{-3}}$)	1.2	2.31×10^{-3}	2.38×10^{-3}
ICWM	0.8	0.981	2.65×10^{6}
ICWM	1.2	2.20×10^{6}	3.98×10^{6}
PEC ICWM	Any	∞	∞

Given the last statement, it is useful to consider the screening parameter k_α, or, more precisely, the screening length $L_\alpha = 2\pi/\mathrm{Re}\,(k_\alpha)$. This screening length is different from the Debye length in the sense that it refers to screening of the total field rather than the incident field (see the appendix). Since the relation between displacement field and electric field involves the total fields, this screening length is relevant for the displacement field and consideration of the ENZ condition. Table 1 shows the two screening lengths for some natural and artificial materials: gold at two frequencies, a SC at two frequencies and for two doping levels, and an isotropic wire medium, where the Debye screening length is defined in the appendix. For the wire medium examples, L_α is quite large and spatial dispersion effects are important (i.e. the relation between displacement field and electric field is strongly nonlocal, and the nonlocal contribution decays algebraically rather than exponentially). However, for metals and SCs the screening length L_α is quite small and spatial dispersion effects are relatively unimportant (the ENZ condition can be approximately satisfied regardless of field polarization).

For the SC we assumed $T = 300\,\mathrm{K}$, $m = 0.26m_e$, $\varepsilon_h = 12$. For gold, $N = 5.9\times10^{28}(\mathrm{m^{-3}})$, and $v_F = 1.4\times10^6\,\mathrm{m\,s^{-1}}$. For the ICWM we used $\varepsilon_h = 1$, $a = \lambda_0/10$, and $r_w = a/100$. For the imperfectly conducting wires, $\varepsilon_m = 1 - \omega_m^2/(\omega(\omega - \mathrm{j}\Gamma))$ where $\omega_m = 1.37\times10^{16}\,\mathrm{s^{-1}}$ and $\Gamma = 5\times10^{13}\,\mathrm{s^{-1}}$.

It can be seen that the wire medium (having a large screening length L_α) cannot realize the ENZ condition excepting either the first or second scenarios described above (e.g. transverse electromagnetic or TE waves). In fact, by looking carefully at previous successful attempts for realizing the ENZ condition using a wire medium, one can see that the third scenario was not implicated. For example, in [22] only a perpendicular electric field exists (which is the first scenario). However, if one tries to realize an ENZ section inside a waveguide carrying a transverse magnetic (TM) wave (third scenario), the wire medium will not be able to completely provide the ENZ condition. To validate this, we consider an x-band rectangular waveguide as shown in figure 2, with height $b = 1.016\,\mathrm{cm}$ and width $a = 2.286\,\mathrm{cm}$. The waveguide has total length 15 cm, with a 5 cm section at each end filled with a simple dielectric having $\varepsilon = 1.5$, and a 5 cm long center region containing an ENZ medium. The frequency of operation is 16.5 GHz, well above cutoff to allow both TE and TM modes to propagate. All simulations were performed using CST Microwave Studio[2].

[2] CST Microwave Studio, www.cst.com.

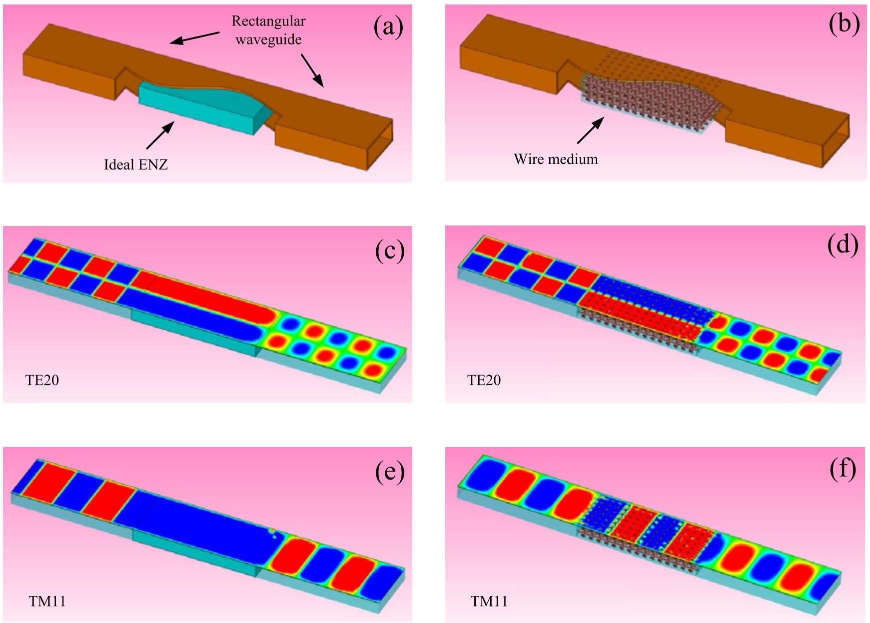

Figure 2. Waveguide with a section of ENZ material: Panel (a) depicts an idealized ENZ material in the center region of the waveguide, otherwise filled with a simple constant-permittivity medium, and panel (b) shows the section filled with an isotropic wire medium. (c) TE_{20} mode in the idealized ENZ, (d) TE_{20} mode in the actual wire medium, (e) TM_{11} mode in the idealized ENZ and (f) TM_{11} mode in the actual wire medium. See movies 1–4 in the supplementary data (available at stacks.iop.org/NJP/15/123027/mmedia).

Figure 2(a) depicts the waveguide with a section of idealized ENZ material (with the permittivity set to $\varepsilon_h = 0.1$), and figure 2(b) shows the section filled with an isotropic wire medium. The host material of the wire medium has $\varepsilon_h = 8$, and the wire period and radius are 3.2 and 0.5 mm, respectively. At the operating frequency, for a local wire-medium plasma model this would result in a ENZ condition (the plasma frequency is 16.7 GHz).

Figure 2(c) shows the result for the TE_{20} mode for the idealized ENZ material. It can be seen that the material is acting like an ENZ material, since the wavelength in the center section is much longer that in the two ends (where $\varepsilon_h = 1.5$). Figure 2(d) shows the corresponding result when the wire medium fills the center region. As expected for a purely transverse TE mode (scenario 1 above), the wire medium also acts as an ENZ material for the TE mode. Figure 2(e) shows the result for the idealized ENZ material for the TM_{11} mode, where, as expected, wavelength in the center section is much longer that in the two ends, similar to the TE case. Finally, figure 2(f) shows the corresponding result for the TM mode for the actual isotropic wire medium. It is evident that the material does not function as an ENZ medium for the TM mode. Given that the TM mode has both transverse and longitudinal components, it corresponds to scenario 3 above (a combination of scenarios 1 and 2). Because the wire medium does not produce a small screening length L_α (see table 1), and because the longitudinal field is not negligible compared to the transverse field, for the TM mode the wire medium does not produce the ENZ condition. However, for a waveguide filled with a SC we would expect to see the ENZ condition for both TE and TM modes (however, the simulation is not possible in CST).

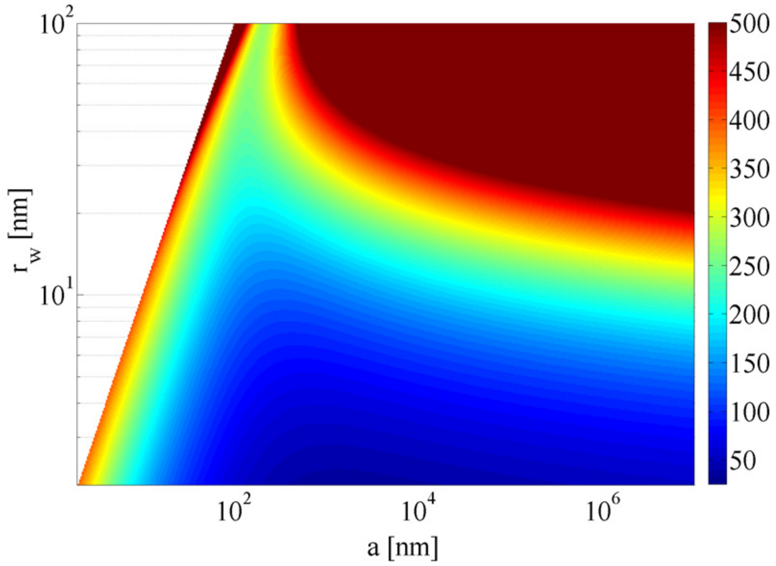

Figure 3. Normalized screening length of the ICWM (L_α/λ_0) as a function of the wire period (a) and radius (r_w) at $\omega = 1.2\omega_p$.

Based on table 1, ICWM cannot be used to realize the ENZ condition for an arbitrary excitation. Of course, the screening length of the ICWM in table 1 is obtained for specific wire parameters (such as wire period and radius). It can be shown that even with changing the parameters of the ICWM, the screening length is extremely large. To clarify this, figure 3 shows the normalized screening length of an ICWM as a function of the wire period (a) and radius (r_w). The wire material is the same as the imperfectly conducting ICWM in table 1, and the frequency is $\omega = 1.2\omega_p$. As figure 3 shows, the normalized screening length is very large, which differentiates the ICWM from the NM described in the table.

4. Conclusion

The 'space-domain' nonlocal permittivity $\underline{\varepsilon}\,(\mathbf{r}-\mathbf{r}')$ has been obtained and a new characteristic screening length introduced for spatially dispersive materials, including artificial wire media. Unlike the Debye length, the new characteristic length relates polarization to the total electric field inside the material, and so it can be used to study the electric displacement distribution in relation to the ENZ condition. Using some typical values SC and wire medium metamaterials, it was shown that the new characteristic length is very small for SCs and therefore an ENZ condition can be easily obtained. However, for wire media the ENZ condition cannot be identically obtained except in some special cases, and often only the perpendicular displacement field can vanish.

Appendix

A.1. Polarization as a function of the scattered field

In the main text we obtain the relationship between polarization and the total electric field. In this and the following appendix, for completeness and because it leads to the important concept of the Debye length, we present the relationship between polarization and the incident electric field, and similarly for the scattered electric field.

An incident field $\mathbf{E}^{\mathrm{i}}(\mathbf{r})$ polarizes the medium, creating polarization current, $j\omega\mathbf{P}(\mathbf{r})$, such that the scattered field $\mathbf{E}^{\mathrm{sca}}(\mathbf{r})$ produced by the induced polarization is

$$\mathbf{E}^{\mathrm{sca}}(\mathbf{r}) = -\mathrm{j}\omega\mu_0 \left(\underline{\mathbf{G}}(\mathbf{r}) * \mathrm{j}\omega\mathbf{P}(\mathbf{r})\right), \tag{A.1}$$

where $\overline{\mathbf{G}}(\mathbf{r})$ is the electric dyadic Green's function [21]. Taking the Fourier transform of (A.1) and using the transform-domain Green's function,

$$\begin{aligned}\mathbf{E}^{\mathrm{sca}}(\mathbf{r}) &= \frac{k^2}{\varepsilon_0}\frac{\underline{\mathbf{I}}k^2 - \mathbf{qq}}{k^2(q^2-k^2)} \cdot \mathbf{P}(\mathbf{q}) \\ &= \frac{k^2\mathbf{P}(\mathbf{q})}{\varepsilon_0(q^2-k^2)} - \frac{q^2\mathbf{P}_{\parallel}(\mathbf{q})\hat{\mathbf{q}}}{\varepsilon_0(q^2-k^2)}.\end{aligned} \tag{A.2}$$

Therefore,

$$\mathbf{P}_{\parallel}(\mathbf{q}) = -\varepsilon_0\mathbf{E}^{\mathrm{sca}}_{\parallel}(\mathbf{q}), \tag{A.3}$$

$$\mathbf{P}_{\perp}(\mathbf{q}) = \frac{\varepsilon_0}{k^2}(q^2-k^2)\mathbf{E}^{\mathrm{sca}}_{\perp}(\mathbf{q}). \tag{A.4}$$

In the space domain, (A.3) and (A.4) become

$$\mathbf{P}_{\parallel}(\mathbf{r}) = -\varepsilon_0\mathbf{E}^{\mathrm{sca}}_{\parallel}(\mathbf{r}) \tag{A.5}$$

$$\mathbf{P}_{\perp}(\mathbf{r}) = -\varepsilon_0\left(1 + \frac{\nabla^2}{k^2}\right)\mathbf{E}^{\mathrm{sca}}_{\perp}(\mathbf{r}). \tag{A.6}$$

Therefore, the longitudinal polarization is local to the longitudinal scattered field, although the transverse polarization is nonlocal to the transverse scattered field.

A.2. Polarization as a function of the incident field

By decomposing the total electric field into the scattered and incident fields as $\mathbf{E}(\mathbf{r}) = \mathbf{E}^{i}(\mathbf{r}) + \mathbf{E}^{\mathrm{sca}}(\mathbf{r})$ and using (15), (16) and (A.3), (A.4),

$$\mathbf{P}_{\parallel}(\mathbf{q}) = \frac{\varepsilon_0}{\varepsilon_{\mathrm{h}}}\left(\varepsilon_{\mathrm{h}} - 1 + \frac{\kappa}{\kappa - \epsilon_{\mathrm{h}}}\frac{k_{\mathrm{D}}^2}{q^2 + k_{\mathrm{D}}^2}\right)\mathbf{E}^{\mathrm{i}}_{\parallel}(\mathbf{q}), \tag{A.7}$$

$$\mathbf{P}_{\perp}(\mathbf{q}) = \varepsilon_0\left(\varepsilon_{\mathrm{h}} - 1 - \kappa\right)\left(1 + \frac{(\varepsilon_{\mathrm{h}} - \kappa - 1)\,k^2}{q^2 - (\varepsilon_{\mathrm{h}} - \kappa)\,k^2}\right)\mathbf{E}^{\mathrm{i}}_{\perp}(\mathbf{q}) \tag{A.8}$$

in the transform domain, where

$$k_{\mathrm{D}} = \alpha\sqrt{\frac{\kappa}{\varepsilon_{\mathrm{h}}} - 1} = \sqrt{\frac{\mathrm{j}\omega\epsilon_0\epsilon_{\mathrm{h}} + \sigma}{D\epsilon_0\epsilon_{\mathrm{h}}}} \tag{A.9}$$

is the Debye wavenumber (usually defined for $\omega = 0$). In the space domain,

$$\mathbf{P}_{\parallel}(\mathbf{r}) = \frac{\varepsilon_0}{\varepsilon_{\mathrm{h}}}\left((\varepsilon_{\mathrm{h}} - 1)\,\mathbf{E}^{\mathbf{i}}_{\parallel}(\mathbf{r}) + \frac{k_{\mathrm{D}}^2\kappa}{\kappa - \epsilon_{\mathrm{h}}}\mathbf{E}^{\mathbf{i}}_{\parallel}(\mathbf{r}) * \frac{\mathrm{e}^{-k_{\mathrm{D}}r}}{4\pi r}\right), \tag{A.10}$$

$$\mathbf{P}_{\perp}(\mathbf{r}) = \varepsilon_0\left(\varepsilon_{\mathrm{h}} - 1 - \kappa\right)\mathbf{E}^{\mathbf{i}}_{\perp}(\mathbf{r}) + \varepsilon_0\left(\varepsilon_{\mathrm{h}} - \kappa - 1\right)^2 k^2 \left(\mathbf{E}^{\mathbf{i}}_{\perp}(\mathbf{r}) * \frac{\mathrm{e}^{-\mathrm{j}k\sqrt{\varepsilon_{\mathrm{h}} - \kappa}r}}{4\pi r}\right). \tag{A.11}$$

These show that the longitudinal polarization has a term that is local with the incident field, as well as a nonlocal contribution that decays according to the Debye length $L_{\mathrm{D}} = 2\pi/k_{\mathrm{D}}$. In the absence of spatial dispersion ($D = 0$), $L_{\mathrm{D}} \to 0$ and the relation between longitudinal polarization and the longitudinal incident field is local. The Debye wavenumber and the new screening wavenumber are related as

$$k_{\mathrm{D}} = k_{\alpha}\sqrt{1 - \frac{\kappa}{\epsilon_{\mathrm{h}}}}.$$

References

[1] Alu A, Engheta N, Erentok A and Ziolkowski R W 2007 *IEEE Antennas Propag. Mag.* **49** 23
[2] Brown J 1953 *Proc. Inst. Electr. Eng.* **100IV** 51
[3] Silveirinha M G and Engheta N 2006 *Phys. Rev. Lett.* **97** 157403
[4] Silveirinha M G and Engheta N 2007 *Phys. Rev.* B **76** 245109
[5] Alu A, Silveirinha M G, Salandrino A and Engheta N 2007 *Phys. Rev.* B **75** 155410
[6] Engheta N 2007 *Science* **317** 1698
[7] Bilotti F, Tricarico S and Vegni L 2008 *New J. Phys.* **10** 115035
[8] Bilotti F, Tricarico S and Vegni L 2010 *IEEE Trans. Nanotechnol.* **9** 55
[9] Navarro-Cía M, Beruete M, Campillo I and Sorolla M 2011 *Phys. Rev.* B **83** 115112
[10] Forati E and Hanson G W 2013 *IEEE AP-S Int. Symp. on Antennas and Propagation* (Orlando, FL: IEEE) at press
[11] Demetriadou A and Pendry J B 2008 *J. Phys.: Condens. Matter* **20** 295222
[12] Pendry J B, Holden A J, Robbins D J and Stewart W J 1998 *J. Phys.: Condens. Matter* **10** 4785
[13] Pendry J B, Holden A J, Stewart W J and Youngs I 1996 *Phys. Rev. Lett.* **76** 4774
[14] Silveirinha M G and Fernandes C A 2005 *IEEE Trans. Microw. Theory Tech.* **53** 1418
[15] Belov P A, Marquès R, Maslovski S I, Nefedov I S, Silveirinha M G, Simovski C R and Tretyakov S A 2003 *Phys. Rev.* B **67** 113103
[16] Halevi P 1992 *Spatial Dispersion in Solids and Plasmas* (Amsterdam: North-Holland)
[17] Silveirinha M G 2007 *Phys. Rev.* B **75** 115104
[18] Hanson G W, Forati E and Silveirinha M G 2012 *IEEE Trans. Antennas Propagation* **60** 4219
[19] Portis A M 1978 *Electromagnetic Fields: Sources and Media* (New York: Wiley)
[20] Silveirinha M G 2009 *Phys. Rev.* B **79** 035118
[21] Chew W C 1995 *Waves and Fields in Inhomogeneous Media* (New York: IEEE Press)
[22] Zhou R, Zhang H and Xin H 2010 *IEEE Trans. Antennas Propagation* **58** 79