Nonlocal Effects on Surface Plasmon Polariton Propagation in Graphene Nanoribbons

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Abstract—Fundamental properties of surface plasmon polariton modes propagating along graphene nanoribbons are investigated by means of a full-wave method-of-moments approach adopting a spatially dispersive dyadic model for graphene conductivity. Modal propagation, current profiles, field distributions, and characteristic impedances are studied in detail showing the dramatic effects of spatial dispersion and the need to go beyond the usual weak (low-q) spatial dispersion formulation.

Index Terms—Graphene, nanoribbons, surface plasmon polariton.

I. INTRODUCTION

PLANAR electromagnetic structures incorporating graphene layers have received considerable attention in the last few years for waveguide, antenna, and shielding applications. In particular, graphene nanoribbons (GNRs) have been proposed in various configurations, as possible interconnects in integrated circuits [1]–[3]. GNRs, either free-standing or placed in a stratified dielectric structure, support a variety of propagation modes, including surface-plasmon modes [4], [5]. Surface plasmon polariton (SPP) propagation in graphene nanostructures has recently attracted interest for the possibility of strong confinement of electromagnetic energy at subwavelength scales, tuned and controlled by a gate voltage or through chemical doping. Other applications of graphene plasmons include optical signal processing, light modulation, sensing, spectral photometry, quantum optics, and nonlinear photonics [6], [7].

Here, we aim at studying the fundamental properties of the dominant SPP mode supported by a monolayer GNR placed on top of a dielectric substrate, employing a rigorous full-wave non-Galerkin method-of-moments (MoM) approach in the spectral domain: the non-Galerkin characteristic (i.e., test functions different from the basis functions) has been chosen to ensure convergence of the spectral integrals defining the MoM matrix. The GNR is modeled through an appropriate tensor surface-impedance boundary condition which takes into account the possible presence of static bias fields and also includes spatial-dispersion effects: this means that graphene is represented through a dyadic conductivity whose elements depend also on the spectral wavenumbers [8], [9]. It is worth noting that the effects of spatial dispersion on the electromagnetic properties of different graphene structures have recently been studied mainly in connection with infinite graphene-sheet structures [10]–[13] and it has been shown that they can dramatically change device performance. Although many studies have been devoted to SPP propagation in GNRs [4], [5], [14], to the best of our knowledge, none has fully addressed the issue of nonlocality (a preliminary investigation on nonlocal effects in GNRs has been presented in [15], but the authors used a low-q model that we will show below to be insufficient for correct modeling).

In the present study, we investigate how spatial dispersion affects the propagation properties along a GNR giving particular attention to the dispersion curves, current profiles, field distributions, and characteristic impedances of the fundamental SPP mode.

It is worth mentioning that, in the more general framework of using GNRs in graphene-based devices and circuits, two critical aspects are SPP excitation and graphene-metal contacts. As concerns the excitation problem, which is a hot research topic, recent studies demonstrated excitation and detection of SPPs in graphene through near-field microscopy nanopits [16], [17]; however, the low excitation efficiency of such near-field sources has prompted research on alternative approaches based on free-space electromagnetic waves. In this case the main difficulty arises from the large momentum mismatch between plasmons and incoming waves, but different techniques have already been successfully developed in this connection, e.g., attenuated total reflection (ATR), scattering from a topological defect at the conductor surface, and Bragg scattering using diffraction gratings or a periodic corrugation of the surface of the conductor [18]–[20]. Although these techniques have been proposed in connection with infinite graphene sheets as well as arrays of GNRs, they may presumably be considered also for the excitation of a single GNR.

As concerns metal-graphene contacts, these constitute a potential limiting factor, especially in active devices, because of the possible occurrence of very large contact resistances, due to several physical and technological reasons, e.g., the low density of states in graphene compared with that of metals, the formation of dipoles and defects at the interface, etc. [21].
Both the considered topics, although important, are beyond the scope of the present investigation.

II. DESCRIPTION OF THE PROBLEM

The electromagnetic problem under analysis is sketched in Fig. 1. It consists of a graphene sheet of width $w$ along the $x$-direction (i.e., a GNR) deposited on a laterally infinite dielectric substrate of thickness $d$ along the $z$-direction and relative permittivity $\varepsilon_r$. All of the units are in the SI system and a time-harmonic variation $e^{j\omega t}$ is assumed and suppressed throughout. Propagation along the $y$-direction, i.e., modes with a spatial dependence $e^{-j\hat{q}_y y}$, is also assumed.

A. Graphene Conductivity

In general, a graphene sheet can be modeled as a conductive sheet with a dyadic surface conductivity [22]

$$ \sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}. $$

In the absence of magnetic bias and neglecting spatial dispersion, graphene can be simply characterized by a scalar local conductivity $\sigma$, which depends on external and internal parameters, e.g., radian frequency $\omega$, temperature $T$, a phenomenological scattering rate $\Gamma = 1/\tau_s$ (where $\tau_s$ is the relaxation time depending on a variety of factors and determined experimentally), and the chemical potential $\mu_c$ (which can be controlled either through doping or through an applied bias electric field orthogonal to the graphene plate) [8]. Moreover, for the considered frequency range (i.e., below hundreds of THz), only the intraband contributions are considered, the interband terms being negligible [8]. In this case

$$ \sigma(\omega) = \frac{\pi}{\alpha} $$

where

$$ \alpha = \omega - j/\tau_s, $$

and $q_e$ is the electron charge, $v_F \approx 10^6$ m/s is the Fermi velocity in graphene, while $\hbar$ and $k_B$ are the reduced Planck and the Boltzmann constants, respectively.

When a more refined model is considered which takes into account spatial dispersion, the graphene conductivity has a dyadic form, whose elements have been derived in [8] under a low-$q$ relaxation-time approximation (RTA) and, more recently, in [9] for arbitrary $q$ values and with the Bhatnagar–Gross–Krook (BGK) model (which allows for including charge diffusion and deriving the correct quantum capacitance). For completeness, the expressions for the dyadic elements for different graphene conductivity models are reported in the Appendix.

B. Dispersion Analysis

Starting from the boundary condition $\vec{E}_0(q_x, q_y, z - h) = \sigma^{-1}(q_x, q_y) \cdot \vec{J}_S(q_x, q_y)$ on the GNR surface, the electric field integral equation (EFIE) can be expressed in the form

$$ \int_{-\infty}^{+\infty} \int_{-w/2}^{+w/2} G^{EJ}(x - x', y - y', z - z' - h) \cdot \vec{J}_S(x', y') \, dx' \, dy' = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \vec{E}(q_x, q_y) \cdot \vec{J}_S(q_x, q_y) \, e^{-j(q_x x + q_y y)} \, dq_x \, dq_y $$

(4)

for $x \in (-w/2, w/2)$ and for any $y$, where $G^{EJ}(\cdot)$ is the EJ-type dyadic Green function for planar layered media, $\vec{J}_S(q_x, q_y)$ is the Fourier transform of the modal surface current $\vec{J}_S(x, y)$, and $\sigma = \sigma^{-1}$ is the dyadic resistivity. By introducing the spectral-domain Green’s function $G^{EJ}(q_x, q_y)$ and representing the surface current of the mode as

$$ \vec{J}_S(x, y) = |J_S(x)| \vec{u}_x + |J_y(x)| \vec{u}_y \cdot e^{-j\hat{q}_y y} = J_S(x) e^{-j\hat{q}_y y} $$

(5)

where $\hat{q}_y = \beta - j\alpha$ is the assumed propagation constant, after some manipulations, (4) can be rewritten as

$$ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[ G^{EJ} (q_x, \hat{q}_y) - \vec{P} (q_x, \hat{q}_y) \right] \cdot \vec{J}_S(q_x) e^{-j\hat{q}_y y} \, dq_x = 0 $$

(6)

for $x \in (-w/2, w/2)$, where $\vec{J}_S(q_x)$ is the Fourier transform of the modal surface current $\vec{J}_S(x)$.

Equation (6) is clearly a nonstandard eigenvalue equation: it has an infinite number of solutions, each of which identifies a propagation mode of the nanoribbon through its propagation constant $\hat{q}_y$, and its surface current distribution $\vec{J}_S(x)$.

C. Numerical Solution

A standard MoM procedure can be used to solve the eigenvalue problem.

In particular, by expanding the $x$ and $y$ components of the modal current in $M$ and $N$ entire-domain (e.g., sinusoidal) basis functions $J_{x,m}$ and $J_{y,n}$, respectively, as

$$ J_{Sx}(x) = \sum_{m=1}^{M} A_m J_{x,m}(x) $$

(7a)

$$ J_{Sy}(x) = \sum_{n=0}^{N-1} B_n J_{y,n}(x) $$

(7b)

where $A_m$ and $B_n$ are unknown coefficients and

$$ J_{x,m}(x) = \sin \left( \frac{2\pi m}{w} x \right), \quad m = 1, 2, \ldots, M $$

(8a)

$$ J_{y,n}(x) = \cos \left( \frac{2\pi n}{w} x \right), \quad n = 0, 1, \ldots, N - 1 $$

(8b)
It should be observed that the Fourier transforms of the basis functions in (8) are
\[
\tilde{J}_{x,m}(q_x) = 4\pi j w^m \frac{(-1)^m \sin \left(\frac{q_x w}{2}\right)}{(q_x w)^2 - (2m\pi)^2}, \quad (9a)
\]
\[
\tilde{J}_{y,n}(q_x) = 2w^2 \frac{(-1)^n q_x \sin \left(\frac{q_x w}{2}\right)}{(q_x w)^2 - (2n\pi)^2}. \quad (9b)
\]
By applying a non-Galerkin test procedure with test functions
\(T_{p,i}(x) = x, y, (i = x, y)\), (6) can be recast in the following matrix form:
\[
\begin{bmatrix}
\tilde{Z}_{xx}^m(q_y) & \tilde{Z}_{xy}^m(q_y) \\
\tilde{Z}_{yx}^m(q_y) & \tilde{Z}_{yy}^m(q_y)
\end{bmatrix}
\begin{bmatrix}
A_m \\
B_n
\end{bmatrix}
= 0 \quad (10)
\]
where \(p = 1, \ldots, M, l = 0, \ldots, N - 1\), and
\[
\tilde{Z}_{rs}^{nm}(q_y) = \int_{-\infty}^{+\infty} \tilde{T}_{r,h}(q_x) \left[ G_{rs}^{EJ}(q_x, q_y) - \rho_{rs}(q_x, q_y) \right] \tilde{J}_{s,q}(q_x) dq_x \quad (11)
\]
where \(r, s, x, y, h = p, l, q = m, n, \) and \(\rho_{rs}\) are the components of the dyadic resistivity \(\rho\). In particular, suitable test functions need to be chosen in order to ensure the convergence of the spectral integral in (11) [23]. This is not trivial, since it can be shown, based on the expressions derived in the Appendix, that in a full-\(q\) formulation we have
\[
\rho_{xx} = \mathcal{O}(q_x^2); \quad \rho_{xy} = \mathcal{O}(q_x) \quad (12)
\]
Hence, if the classical Galerkin test procedure were used, the integrand functions of the matrix elements \(\tilde{Z}_{rs}^{nm}(q_y)\) would show an asymptotic behavior as \(q_x^{-1}\) without alternating sign thus leading to non-convergent integrals.

A suitable choice of test functions is instead
\[
T_{x,p}(x) = \sin \left(\frac{2\pi p x}{u} \right) \quad p = 1, 2, \ldots, M \quad (13a)
\]
\[
T_{y,l}(x) = \cos \left[ \frac{2\pi (l + 1)x}{u} \right] + (-1)^l \quad l = 0, 1, \ldots, N - 1 \quad (13b)
\]
The Fourier transform of (13a) is the same as in (9a), and it has an asymptotic behavior as \(\mathcal{O}(q_x^{-2})\), while the Fourier transform of (13b) is
\[
\tilde{J}_{y,l}(q_x) = 8\pi^2 \frac{(-1)^l l^2 \sin \left(\frac{q_x w}{2}\right)}{q_x^3 w^2 - 4q_x^2 l^2 \pi^2} \quad (14)
\]
with an asymptotic behavior as \(\mathcal{O}(q_x^{-2})\) which ensures the convergence of all of the integrals (11).

The computation of the matrix elements in (11) involves the integration of functions which oscillate as \(\sin^2 (q_x w/2)\): the integration can then efficiently be performed through the double exponential formulas proposed in [24].

The eigenvalues \(\hat{\omega}_l\) have to be found searching for the complex zeroes of the determinant of the square \(M + N\) matrix in (10), i.e.,
\[
\det [\mathbf{Z}(\hat{\omega}_l)] = 0 \quad (15)
\]
and a dispersion analysis \(\hat{\omega}_l(f)\) can easily be performed tracking the relevant complex pole as a function of the frequency \(f\).

Although several method are available for searching zeros on the complex plane [25], [26], the Müller’s method is here employed since it is simple and robust. Since the number of iterations required to converge to the correct propagation constant (matrix eigenvalue) \(\hat{\omega}_l\) strictly depends on the initial guess, a suitable predictor has been developed based on an extrapolation procedure of the previous ten eigenvalues found at the previous frequencies. It results in a very efficient code that is able to track the eigenvalue at each frequency step with just three or four iterations.

D. Current, Field, and Characteristic Impedance Calculation

Once the propagation constant \(\hat{\omega}_l\) that makes null the determinant of the MoM matrix at a prescribed frequency is found, the eigenfunctions associated with the matrix \(\tilde{\mathbf{Z}}(\hat{\omega}_l)\) can easily be computed and the eigenfunction corresponding to the minimum eigenvalue (which is null within the machine accuracy) gives the expansion coefficients in (7); this way, both the longitudinal and transverse current components can easily be computed.

Once the modal current \(J_S(x, y)\) on the GNR surface is known, the modal field components can then be computed as
\[
E_r(x, y, z) = \frac{e^{-j\omega_\varphi}}{2\pi} \int_{-\infty}^{+\infty} \tilde{G}_{EJ}(q_x, q_y; z, \hat{\omega}_l(q_x)) J_S(q_x) d\omega_\varphi \quad (16a)
\]
\[
H_r(x, y, z) = \frac{e^{-j\omega_\varphi}}{2\pi} \int_{-\infty}^{+\infty} \tilde{G}_{HJ}(q_x, q_y; z, \hat{\omega}_l(q_x)) J_S(q_x) d\omega_\varphi \quad (16b)
\]
where \(r = \{x, y, z\}\) while \(\tilde{G}_{EJ}\) and \(\tilde{G}_{HJ}\) are the EJ- and HJ-type spectral-domain dyadic Green’s functions, respectively [27]. It is worth noting that the computation of all of the involved integrals can be made more efficient by suitably exploiting the symmetries of the integrand functions.

The characteristic impedance is then calculated according to the current-power definition [28]
\[
Z_c = \frac{P}{\bar{I}^2} = \frac{\int_{-\infty}^{+\infty} \int_{w/2}^{+\infty} \mathbf{E}(r) \cdot \mathbf{H}^*(r) \cdot u_y \, dz \, dz}{\int_{w/2}^{+\infty} \int_{w/2}^{+\infty} J_y(x) \, dx \, dz} \quad (17)
\]
where the integral in the numerator of (17) is recursively computed over an increasing surface centered on the GNR and truncated when the prescribed accuracy is reached.

III. RESULTS

A. Graphene Conductivity and Basic Structure

Here, a comparison among the different graphene conductivity models is presented for different radial frequencies \(\omega\), assuming in all cases \(\tau_c = 0.5\) ps, \(\mu = 0\) eV, and \(T = 300\) K. The conductivity dyadic will be represented in polar coordinates, showing results for the longitudinal and transverse conductivities \(\sigma_L\) and \(\sigma_T\) as functions of the radial wavenumber \(q\) normalized with respect to the free-space wavenumber \(k_0\) [37].
For clarity, we summarize the different models considered. For the RTA formulations there are two conductivity expressions: the full-\( q \)-expression and the low-\( q \)-approximation (cf. (33)-(34) and (28)-(29), respectively). For the BGK formulation, which is more accurate than the RTA, we also have two conductivity expressions: the full-\( q \)-expression and the low-\( q \)-approximation (cf. (30)-(31) and (26)-(27), respectively). We also show the scalar local model (2). Similar results have already been shown in [9], but we report them here for completeness and then focus on nonlocal GNR modal properties which have not been examined in detail.

In Fig. 2, the absolute values of \( \sigma_\| \) and \( \sigma_\perp \) are shown as functions of \( q/k_0 \) at \( f = 1 \) THz.

![Graph showing \( |\sigma_\|| \) as a function of \( q/k_0 \) at \( f = 1 \) THz.](image)

![Graph showing \( |\sigma_\perp| \) as a function of \( q/k_0 \) at \( f = 1 \) THz.](image)

In order to show the effects of spatial dispersion on the electromagnetic properties of GNRs, we consider a GNR structure with \( w = 200\) nm (more than two orders of magnitude larger than the graphene lattice constant \( a_s \), so that electronic edge-effects can be ignored) and \( d = 400\) nm, with different permittivity values for the substrate. All of the results have been obtained with six basis functions for each current component (i.e., \( M = N = 6 \)) which provide converged results.

### B. SPP Propagation Characteristics

In Fig. 3, the dispersion properties of the fundamental SPP mode supported by a free-standing GNR (i.e., \( \varepsilon_r = 1 \)) are reported in a logarithmic scale. In particular, the normalized phase \( \beta/k_0 = \text{Re} \{ q_y/k_0 \} \) and attenuation \( \alpha/k_0 = -3\pi \text{Im} \{ q_y/k_0 \} \) constants are plotted as functions of frequency by adopting three different conductivity models for graphene, i.e., a local formulation, a low-\( q \) BGK formulation, and a full-\( q \) BGK formulation. The latter fully takes into account the nonlocal effects which are seen to be particularly pronounced in the lowest (microwave) frequency range. On the other hand, as expected, the low-\( q \) formulation leads to erroneous results especially in the lowest frequency range (this happens both in the RTA and BGK formulations).

In Fig. 4(a) and (b), the normalized phase \( \beta/k_0 \) and attenuation \( \alpha/k_0 \) constants are reported as functions of frequency for a free-standing GNR with different ribbon widths: from Fig. 4, it can thus be seen that nonlocal effects are stronger for ribbons with narrower widths and negligible for ribbons with widths larger than 1 \( \mu \)m.

In Fig. 5(a) and (b), the dispersion properties are reported for a GNR with two different permittivities of the substrate, i.e., a moderate permittivity \( \varepsilon_r = 3.9 \) [Fig. 5(a)] and a high permittivity \( \varepsilon_r = 10.2 \) [Fig. 5(b)]. In these cases the low-\( q \) results have not been reported since it can be shown that such a low-\( q \) formulation dramatically fails. On the other hand, it can easily be seen that increasing the dielectric permittivity strongly enhances the spatial-dispersion effects, as shown in [11] for modes supported by an infinite graphene sheet. In particular, it can be seen that in the high-permittivity case ignoring spatial dispersion leads...
C. Modal Currents and Field Distributions

In Fig. 6, the amplitude of the longitudinal $y$-component $J_y$ of the modal current along the nanoribbon (normalized to its maximum value) is reported at the operating frequency $f = 1$ THz for a structure as in Fig. 3. The transverse $x$-component $J_x$ is not reported since its value is more than three-orders of magnitude smaller than the longitudinal component $J_y$. It can be seen that ignoring spatial dispersion or using a spatially-dispersive model valid for low wave numbers leads to completely erroneous current profiles. In particular, the almost constant behavior typical of the fundamental mode in microwave microstrip line is obtained using a purely local model, whereas the correct spatially dispersive full-$q$ conductivity formulation determines a variable current profile across the ribbon with a minimum at the edges and a maximum near the center of the graphene nanostrip. Such a behavior is exactly the opposite of what is found adopting a low-$q$ model which predicts a current profile far from both the purely local model and the correct spatially dispersive model. Again, using a low-$q$ model in such problems dramatically affects the correctness of the results.

It is worth noting that the behavior shown in Fig. 6 is qualitatively the same across the considered frequency spectrum (from 10 GHz to 10 THz) and for structures with different substrate permittivities.

The normalized electric and magnetic field distributions are reported in Fig. 7(a) and (b), respectively, for a structure as in Fig. 6 (i.e., $\varepsilon_r = 1$ and $f = 1$ THz). They show a classical 2-D monopole-like behavior and have been obtained using the full-$q$ conductivity formulation. A local model would lead to the same qualitative field behavior: to observe the quantitative differences, in Fig. 8(a) the amplitude of the normalized electric fields are reported as functions of $z$ for $x = 0$ at $f = 1$ THz.

Fig. 4. Normalized propagation constants as functions of frequency $f$ for a structure as in Fig. 1 with $\varepsilon_r = 1$ for different ribbon widths. Two different conductivity models are considered, i.e., a local model and a full-$q$ BGK model for the calculation of (a) the normalized phase constant $\beta/k_0$ and (b) the normalized attenuation constant $\alpha/k_0$.

Fig. 5. Normalized phase ($\beta/k_0$) and attenuation ($\alpha/k_0$) constants as functions of frequency $f$ for a structure as in Fig. 1 with (a) $\varepsilon_r = 3.9$ and (b) $\varepsilon_r = 10.2$. Two different conductivity models are considered, i.e., a local model and a full-$q$ BGK model.
Fig. 6. Normalized $y$-component of the modal current $J_y$ across the nanoribbon for a structure as in Fig. 1 with $\varepsilon_r = 1$ and $f = 1$ THz. Different conductivity models are considered: a local model, a low-$q$ BGK model, and a full-$q$ BGK model.

Fig. 7. Normalized (a) electric and (b) magnetic field distribution of the plasmon mode for a structure as in Fig. 6 using a full-$q$ BGK model.

for a free-standing GNR. In order to point out the spatial-dispersion effects in the modal field distributions of GNRS the same of Fig. 8(a) is reported in Fig. 8(b) for a structure with $\varepsilon_r = 10.2$ and at $f = 10$ THz. It can be seen that, while in the former case the plasmon mode is poorly confined and spatial-dispersion effects are negligible, in the latter the electric field is well concentrated near the ribbon, although the inclusion of spatial dispersion greatly reduces the amount of field confinement. This also occurs for, e.g., localized surface plasmons, where spatial dispersion reduces the field concentration in the gap between two plasmonic dimers [29].

D. Characteristic Impedance

The real and imaginary parts of the characteristic impedance are reported as functions of frequency in Fig. 9(a) for a free-standing GNR as in Figs. 3 and in Figs. 9(b) for a high-dielectric GNR as in Fig. 5(b). As it can be seen, at low frequencies, both the real and imaginary parts are of the order of $k\Omega$, and there is a significant discrepancy between the local model and the full-$q$ BGK formulation: such a discrepancy disappears beyond 1 THz for the real part and beyond 2 THz the imaginary part become one order of magnitude smaller than the real part. Interestingly, the spatial-dispersion effects on the characteristic impedance of the GNR are weaker in the high-permittivity case.

E. Electrical Conductivity Tuning

As is well known, one of the most attractive characteristic of graphene is the possibility of externally controlling its electrical conductivity by modifying the chemical potential $\mu_c$: this can be obtained either by applying an electrostatic bias or by doping [8]. In a more general framework, such an external tuning could be used to dynamically control the reflection, absorption, or polarization of the electromagnetic waves impinging against a graphene shield [30], to control the radiation characteristics of leaky-wave antennas [31], or to make graphene layers working as tunable cloaks [32]. Moreover, in recent works it has been
shown that graphene can effectively act as a platform for infrared metamaterials and transformation optical devices [33], [34]. For plasmon propagation in GNRs, the conductivity tuning allows for varying the chemical potential across the strip (thus generating a nonhomogeneous conductivity), or for creating different channels for propagation over a uniform sheet [35]. The analysis of inhomogeneous GNRs is out of the scope of the present investigation and it requires different numerical tools [36]; rather, it is here interesting to investigate the effects of a uniform variation of the chemical potential on the propagation features of the dominant plasmon mode in GNRs, i.e., we consider a GNR with a homogeneous conductivity whose value can be tuned at a fixed frequency by externally varying the chemical potential.

In Fig. 10 the normalized phase and attenuation constants are reported as functions of the chemical potential for a structure as in Fig. 1 with $\varepsilon_r = 1$ at the operating frequency $f = 1$ THz. As for the dispersion analysis, two different conductivity models are considered, i.e., a local model and a full-$q$ BGK model. It can be seen that the influence of the chemical potential on the propagation properties of GNRs is the same for the local and the spatially-dispersive model: in fact, a variation of $\mu_c$ only affects the term $\gamma$ in (3). By increasing $\mu_c$, the real part of the conductivity (which is responsible of losses) decreases and, accordingly, the attenuation constant of the propagating plasmon mode also decreases: such a decreasing is more pronounced for small values of $\mu_c$. Although reported in Fig. 10 for a particular configuration of GNRs, such a behavior is general, as verified by numerous simulations.

In Fig. 11, the real and imaginary parts of the characteristic impedance of the same structure as in Fig. 10 are reported as functions of the chemical potential. These results have been obtained using a local model and a full-$q$ BGK formulation for the graphene conductivity. As can be seen, the variation of the chemical potential can tune the values of the characteristic impedance by reducing it by about 80%.

IV. DISCUSSION AND CONCLUSION

Nonlocal effects that severely affect the propagation properties of the fundamental plasmon mode in graphene nanoribbons have been carefully investigated by means of a Method-of-Moments approach in the spectral domain which fully takes into account the spatially dispersive nature of the intraband graphene conductivity. Results have been presented for the propagation constant, the current profile, the field patterns, and the characteristic impedance of the dominant plasmon mode showing that spatial dispersion have to be taken into account through a model valid for arbitrarily large $q$ value if dramatic errors are to be avoided.
In particular, it has been shown that the strongest nonlocal
effects occur for GNRs with small widths and supported by a
dielectric with high permittivity: usually the nonlocality largely
attenuates the longitudinal propagation of the plasmon mode
and makes it slower at low frequencies and faster at high
frequencies. The transverse profile of the longitudinal component
of the current is strongly modified showing a maximum at the
center of the ribbon, while the modal field is less tightly con-
fined to the ribbon. The characteristic impedance is less affected
by spatial dispersion, at least for high frequencies, whereas in
the low frequency (microwave) regime can be largely increased
both in its resistive and capacitive part. Bias voltages or chem-
ical doping can be effectively used to tune the modal proper-
ties of the propagating SPP and, in particular, the former can
be used to locally modify the transverse conductivity profile to
create a tunable inhomogeneous conductivity (this is left for fu-
ture works).

As concerns the effects that nonlocality may have on
graphene-based devices, the large variety of structures pro-
posed in recent years, e.g., phase-shifters, filters, absorbers,
prevents one from drawing sharp conclusions. However, in
the light of the results obtained in the present investigation,
our feeling is that nonlocality does not necessarily spoil the
expected performance of GNR-based components, but, rather,
it is a key parameter that must be taken into account in proper
device modeling and design.

APPENDIX

For completeness, we report in this Appendix the expres-
sions of the dyadic elements for different graphene conductivity
models derived in [9]. In particular, we show in a low-\(q\) RTA model
\[
\begin{align*}
\sigma_{xx}^{\text{RTA}} &= \frac{\pi}{\alpha} \left(1 + \frac{3\nu_F^2}{4\alpha^2} q_x^2 + \frac{\nu_F^2}{4\alpha^2} q_y^2\right) \\
\sigma_{xy}^{\text{RTA}} &= \frac{\nu_F^2}{2\alpha^2} q_x q_y \\
\sigma_{yx}^{\text{RTA}} &= \frac{\nu_F^2}{2\alpha^2} q_y q_x \\
\sigma_{yy}^{\text{RTA}} &= \frac{\nu_F^2}{4\alpha^2} q_x^2 + \frac{3\nu_F^2}{4\alpha^2} q_y^2
\end{align*}
\]
whereas in a low-\(q\) BGK model we have
\[
\begin{align*}
\sigma_{xx}^{\text{BGK}} &\sim \frac{\pi}{\alpha} \left[1 + \frac{\nu_F^2}{4\alpha^2} \left(3 + \frac{i}{2\omega_T^2}\right) q_x^2 + \frac{\nu_F^2}{4\alpha^2} q_y^2\right] \\
\sigma_{xy}^{\text{BGK}} &\sim \frac{\nu_F^2}{2\alpha^2} \left(1 + \frac{1}{\omega_T^2}\right) q_x q_y \\
\sigma_{yx}^{\text{BGK}} &\sim \frac{\nu_F^2}{2\alpha^2} \left(1 + \frac{1}{\omega_T^2}\right) q_y q_x \\
\sigma_{yy}^{\text{BGK}} &\sim \frac{\nu_F^2}{4\alpha^2} q_x^2 + \frac{\nu_F^2}{4\alpha^2} \left(3 + \frac{i}{2\omega_T^2}\right) q_y^2
\end{align*}
\]
In an RTA model, the dyadic conductivity elements whose
expressions are instead valid for arbitrary values of \(q\) are
\[
\begin{align*}
\mathbf{\sigma}^{\text{RTA}} &= \gamma \mathbf{I}_\phi
\end{align*}
\]
where
\[
\begin{align*}
I_{\phi_{xx}}(q_x, q_y) &= -2\pi \frac{\nu_F^2 q_x^2 q_y^2 R - \alpha \nu_F q_x p^2 - \alpha^2 p^2 (1 - R)}{v_F^2 (\alpha + \nu_F q_x) q^4} \\
I_{\phi_{xy}}(q_x, q_y) &= I_{\phi_{yx}}(q_x, q_y) \\
&= -2\pi q_x q_y \frac{\nu_F^2 q_x^2 R + 2\alpha \nu_F q_x + 2\alpha^2 (1 - R)}{v_F^2 (\alpha + \nu_F q_x) q^4} \\
I_{\phi_{yy}}(q_x, q_y) &= -2\pi \frac{\nu_F^2 q_y^2 R + \alpha \nu_F q_x p^2 + \alpha^2 p^2 (1 - R)}{v_F^2 (\alpha + \nu_F q_x) q^4}
\end{align*}
\]
with \(q^2 = q_x^2 + q_y^2, p^2 = q_x^2 - q_y^2,\) and
\[
R(q_x, q_y) = \frac{\alpha + \nu_F q_x}{\sqrt{\alpha^2 - \nu_F^2 q^2}}.
\]
In a BGK full-\(q\) model, we have
\[
\begin{align*}
\sigma_{xx}^{\text{BGK}} &= \gamma (I_{\phi_{xx}} q_y - I_{\phi_{xy}} q_x) \frac{\Delta(q_x, q_y)}{D_\sigma} \\
\sigma_{xy}^{\text{BGK}} &= \gamma (I_{\phi_{xy}} q_y - I_{\phi_{yx}} q_x) \frac{\Delta(q_x, q_y)}{D_\sigma} \\
\sigma_{yx}^{\text{BGK}} &= \gamma (I_{\phi_{yx}} q_x - I_{\phi_{xx}} q_y) \frac{\Delta(q_x, q_y)}{D_\sigma} \\
\sigma_{yy}^{\text{BGK}} &= \gamma (I_{\phi_{yy}} q_x - I_{\phi_{xy}} q_y) \frac{\Delta(q_x, q_y)}{D_\sigma}
\end{align*}
\]
where
\[
\begin{align*}
\gamma_D &= \frac{\nu_F}{2\pi w_T} \nonumber \quad \text{and} \\
D_\sigma &= 1 + \gamma_D \Delta q^2 \nonumber \quad \text{with}
\end{align*}
\]
\[
\Delta(q_x, q_y) = -\frac{2\pi}{\nu_F q^2} \left(1 - \frac{\alpha}{\sqrt{\alpha^2 - \nu_F^2 q^2}}\right).
\]
Simpler expressions can be obtained in a polar coordinate
system through the conventional longitudinal \(\sigma_L\) and transverse \(\sigma_T\) representation [37]. In particular, in a low-\(q\) approximation
\[
\begin{align*}
\sigma_L^{\text{BGK}} &= \frac{\pi}{\alpha} \left[1 + \left(\frac{3}{4} + \frac{i}{2\omega_T}\right) \frac{\nu_F^2}{\alpha^2} q^2\right] \\
\sigma_T^{\text{BGK}} &= \frac{\pi}{\alpha} \left[1 + \frac{\nu_F^2}{4\alpha^2} q^2\right]
\end{align*}
\]
while
\[
\sigma_L^{\text{RTA}} = \sigma_T^{\text{RTA}} = \frac{\pi}{\alpha} \left[1 + \frac{3\nu_F^2}{4\alpha^2} q^2\right]
\]
and
\[
\sigma_L^{\text{BGK}} = \sigma_T^{\text{BGK}} = \gamma \nu_F \chi \sigma_L^{\text{RTA}} \quad \text{and}
\]
whereas the relevant full-\(q\) expressions are
\[
\begin{align*}
\sigma_L^{\text{BGK}} &= \frac{\nu_F}{2\pi \gamma_D (1 - \chi) + \nu_F \chi \sigma_L^{\text{BGK}}} \\
\sigma_T^{\text{BGK}} &= -\frac{2\pi \gamma_D}{\nu_F q^2} (1 - \chi)
\end{align*}
\]
with
\[
\chi = \sqrt{1 - \nu_F^2 q^2/\alpha^2}
\]
for the BKG model and

$$\sigma_T^{RHA} = \frac{1}{\chi} \sigma_T^{RTA}$$

(33)

$$\sigma_T^{RTA} = \sigma_T^{RHA}$$

(34)

for the RTA model. It should be noted that both $\sigma_T$ and $\sigma_T^r$ always depend only on $q$. This indicates that, in the absence of magnetic bias, graphene is modeled as an isotropic medium, the dyadic nature of its conductivity being uniquely due to the occurrence of spatial dispersion.

REFERENCES


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