Homogenized Green’s Functions for an Aperiodic Line Source Over Planar Densely Periodic Artificial Impedance Surfaces

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Abstract—The accurate electromagnetic analysis of artificial periodic surfaces formed as planar layers with complicated periodic metallization patterns, having a grid period much smaller than the effective wavelength (densely periodic), is important for the design and analysis of a variety of electromagnetic structures. However, full-wave modeling can be extremely time-consuming and computationally expensive, especially for aperiodic sources in close proximity to periodic surfaces. In this paper, we describe approximate homogenized models for a Green’s function that treats planar patterned screens (grids) as quasi-dynamic homogenized impedance surfaces and dielectric layers in a fully dynamic manner. The resulting Green’s functions are only slightly more complicated than those for dielectric layers without metallization and can be numerically computed using standard methods for layered media. We restrict attention to line sources and compare numerical results from this method with those from a full-wave array scanning method, which is more complex analytically and much more demanding to evaluate numerically. Very good agreement is found between the two methods except for source and/or field points extremely close to the metallization layer, confirming the accuracy of the homogenized representations of periodic surfaces for near-field sources.

Index Terms—Array scanning method (ASM), electromagnetic analysis, Green’s functions, high-impedance surfaces, nonhomogeneous media, periodic structures.

I. INTRODUCTION

In recent years, there has been a growing interest in the analysis and development of densely periodic artificial impedance surfaces (with the grid period much smaller than the effective wavelength) due to their broad applications in the emerging areas of metamaterials. In particular, high-impedance surfaces (HISs), originally proposed by Sievenpiper as mushroom-type periodic structures formed by a 2-D square lattice of nonresonant patches with grounding vias [1], have been used as artificial high-impedance textured substrates for low-profile antennas [2]. Other designs of HIS structures utilize subwavelength periodic arrays of frequency-selective surface (FSS) elements of different shapes without vias printed on a thin grounded dielectric slab. This includes patch arrays [3]–[5], printed dipole/slot arrays [6], [7], dipole/slot arrays of different resonant length of FSS elements in the unit cell [8], [9], and complicated configurations of FSS elements [10], among others. Other applications of HIS structures and, in general, artificial periodic impedance surfaces include thin absorbers [11]–[13], planar tunable reflect-arrays [14], [15], TEM waveguides [16], [17], partially reflecting surfaces (PRSs) for high-directivity/gain antennas [18]–[22], and leaky-wave antennas with broadside radiation [23], [24].

An approach for the accurate and rapid analysis of plane-wave interaction with densely periodic planar metallization patterns used in HIS structures has been proposed in [25]–[28], and the analytical model for the analysis of surface-wave and leaky-wave propagation has been presented in [29]. The dynamic model is based on the full-wave solution of a plane-wave scattering problem incorporating an averaged impedance boundary condition and enables one to accurately capture the physics of plane-wave interaction with complicated metallization patterns by modeling a single unit cell of a periodic grid and considering a single Floquet propagating mode. It is based on the homogenization of grid impedance in terms of effective circuit parameters (inductance and capacitance). It should be noted that the analytical expressions for the grid impedance take into account frequency dispersion and spatial dispersion (the latter corresponding to the dependence of the grid parameters on the incidence angle), and they have been obtained by considering the main contribution of all elements of the infinite grid to the local field [26].

However, the exact analysis of the interaction of an aperiodic fundamental near-field source (a line or point source) with periodic metallization patterns requires considerable effort. The array-scanning method (ASM) is an efficient technique for the full-wave analysis of the planar periodic artificial impedance surfaces described above [30], [31], although the formulation is somewhat complex and numerically demanding. In the special case when the metallization is densely periodic, one can hope to use a homogenization theory to simplify the formulation. In fact, this is a regime that is particularly demanding for full-wave methods due to the required dense discretization of the struc-
In this connection, investigations on the limits of validity of classical homogeneous models for the field description in 2-D problems involving a line source and structures periodic in one direction have been previously carried out in [32]–[35].

In this study, we investigate the accuracy of homogenized representations of planar periodic screens, initially proposed and validated for plane-wave incidence (see, for example, [28]) for the calculation of the field excited by an aperiodic line source. In particular, quasi-analytical homogenized Green’s function models are presented for the analysis of planar metallization patterns densely periodic in *two directions*, printed on dielectric substrates. The homogenization analysis makes use of either a single impedance surface composed of the parallel combination of grid and dielectric slab impedances [26] or a two-sided impedance boundary condition for the metallization in the boundary-value problem for the grounded dielectric slab [29]. These two methods lead to the same result, which is compared with results obtained using a full-wave ASM. Very good agreement is found for the simple quasi-analytic approach presented here, except for source and field points approaching extremely closely to the metallization.

We restrict the source to be a line source, which decouples the problem into transverse magnetic (TM) and transverse electric (TE) parts. Since a general point source excites both TE and TM potentials, one requires a spectrally anisotropic dyadic grid impedance (represented by TE and TM parts along with cross-coupling terms); the extension to a point source will be considered in future work. In the following, the suppressed time dependence is $e^{j\omega t}$.

**II. LINE-SOURCE GREEN’S FUNCTIONS: HOMOGENIZED MODELS**

*A. General Formulation*

Consider the geometry depicted in Fig. 1, showing a line source over a layered medium. Each material interface may have a grid impedance $Z_{g,n}$ for the $n$th interface.

Following the method of obtaining the Green’s dyadics in [36], adapted here to the incorporation of impedance surfaces, the fields in the region of space $z > 0$ have the forms [37], [38]

$$
\mathbf{E}(\mathbf{r}) = \left( k_1^2 + \nabla \cdot \right) \mathbf{\pi}_e(\mathbf{r}) - j\omega \mu_1 \nabla \times \mathbf{\pi}_m(\mathbf{r}) \tag{1}
$$

$$
\mathbf{H}(\mathbf{r}) = j\omega \varepsilon_1 \nabla \times \mathbf{\pi}_e(\mathbf{r}) + \left( k_1^2 + \nabla \cdot \right) \mathbf{\pi}_m(\mathbf{r}) \tag{2}
$$

where $k_1^2 = \omega^2 \mu_1 \varepsilon_1$ and $\mathbf{\pi}_{e,m}$ are the electric and magnetic Hertzian potentials associated with electric and magnetic currents $\mathbf{J}_{e,m}$, respectively. Each can be written as [36]

$$
\mathbf{\pi}_{e,m}(\mathbf{r}) = \mathbf{\pi}^p(\mathbf{r}) + \mathbf{\pi}^s(\mathbf{r})
$$

$$
= \int_V \left[ \mathbf{g}^p(r,r') + \mathbf{g}^s(r,r') \right] \cdot \mathbf{P}_{e,m}(r') \cdot \frac{dV'}{j\omega} \tag{3}
$$

where $\mathbf{P}_e = \mathbf{J}_e/\varepsilon_1$, $\mathbf{P}_m = \mathbf{J}_m/\mu_1$, and $\mathbf{g}^p$, $\mathbf{g}^s$ are the direct and scattered dyadic Green’s functions for the Hertzian potentials. The potential $\mathbf{\pi}^p$ is due to the primary wave, incident from the source in a homogeneous medium characterized by $\varepsilon_1, \mu_1$, and $\mathbf{\pi}^s$ is the scattered potential (in this case, reflected potential) that accounts for the layered medium. For source and field points in arbitrary layers, the above formulation is easily generalized.

For a 2-D infinite electric line source

$$
\mathbf{J}_e = \hat{y} I_0 \delta (z - z_0) \delta (x) \tag{4}
$$

we have

$$
\mathbf{E}(x, z) = \frac{I_0 k_1^2}{j\omega \varepsilon_1} \left[ \frac{1}{4\pi} H_0^{(2)}(k_1 \rho) + \frac{1}{2\pi} \int_{-\infty}^{\infty} R_k(k_x) e^{-p(z-z_0)} e^{jk_kx} \frac{1}{2p_1} dk_x \right]
$$

$$
= \frac{I_0 k_1^2}{j\omega \varepsilon_1} \left( \mathbf{\pi}^p + \mathbf{\pi}^s \right) \tag{5}
$$

where $\rho = \sqrt{x^2 + (z-z_0)^2}$, $p_1 = k_2^2 - k_1^2$, and the spectral coefficient $R_k(k_x)$ is associated with $\mathbf{TE}^{p,z}$ potential (a $y$-polarized electric line source creates a $\mathbf{TE}^{p,z}$ field). Here, the first term is associated with the 2-D principal scalar Green’s function

$$
\mathbf{g}^p(r,r') = \frac{1}{4\pi} H_0^{(2)}(k_1 \rho)
$$

$$
= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} e^{-\rho_1 |z-z_0|} e^{jk_kx} \frac{1}{2p_1} dk_x \tag{6}
$$

and the second term is the scattered potential. The latter is obtained by enforcing the appropriate boundary conditions at each interface

$$
\hat{z} \times \left( \mathbf{H}^+ - \mathbf{H}^- \right) = \mathbf{J}_e^p
$$

$$
\hat{z} \times \left( \mathbf{E}^+ - \mathbf{E}^- \right) = \mathbf{0} \tag{7}
$$

where the superscripts indicate a position infinitesimally above and below the interface, and where $\mathbf{J}_e^p$ ($\text{A/m}$) is an electric surface current on the boundary

$$
\mathbf{J}_e^p = \frac{1}{Z_g} \mathbf{E}_e \tag{8}
$$

The units of the grid impedance $Z_g$ are ohms.

For a magnetic line source

$$
\mathbf{J}_m = \hat{y} I_0 \delta (z - z_0) \delta (x) \tag{9}
$$
and, considering the magnetic Hertzian potential, we obtain
\[
\mathbf{H}(x, y) = \frac{j \omega \mu_2}{j \omega \mu_1} \left[ \frac{1}{4\pi} H_0^{(2)}(k_\rho) + \frac{1}{2\pi} \int_{-\infty}^{\infty} R_n(k_\rho) e^{-p_1(z+z_0)} \frac{e^{j k_\rho x}}{2p_1} \, dp_1 \right]
\]
\[
= \frac{j \omega \mu_2}{j \omega \mu_1} \left( \pi^2 + \pi_8^2 \right). \tag{10}
\]

The spectral coefficient \(R_n\) in (10) is associated with TM\(_z\) potential (a \(y\)-polarized magnetic line source creates a TM\(_{x,z}\) field). The coefficients \(R_n\) depend on the layered medium and will be presented in Section II-B.

**B. Green’s Function Coefficients**

Here, two analytical methods for developing Green’s function coefficients \(R_n\) for the analysis of densely periodic metallizations in layered media are described. In both cases, we assume a periodic metallization layer on a grounded dielectric slab, but the method is easily generalized to multiple layers. In the first model, a single impedance surface is obtained in the spectral domain as parallel connection of grid impedance of homogenized metallization and an input impedance for the grounded dielectric slab. The second model is based on the implementation of a two-sided impedance boundary condition for the homogenized metallization in the boundary-value problem for a grounded dielectric slab. Mathematically, the two methods are equivalent. The first method has the advantage that it provides the simplest formulation, however, it is somewhat cumbersome to find the fields in the dielectric (since the grounded dielectric has been subsumed into an input impedance). On the other hand, using the second method it is very easy to directly determine the fields in the dielectric using equations similar to (5) and (10) (but omitted due to space limitations).

1) **Method I: Single Impedance Interface**: Fig. 2 shows the original problem (for the special case of an array of metal patches), and the homogenized problem represented as a single one-sided impedance sheet \(Z_{\mathrm{g}}\).

The surface impedance in the spectral domain is obtained as parallel combination of grid impedance \(Z_{\mathrm{g}}\) that represents a homogenized planar periodic metallization pattern (grid) and the input impedance \(Z_{\mathrm{d}}\) of the grounded dielectric slab,
\[
Z_{\mathrm{g}} = \frac{Z_{\mathrm{g}}^{\mathrm{TE}, \mathrm{TM}}(k_{\rho}) Z_{\mathrm{d}}^{\mathrm{TE}, \mathrm{TM}}(k_{\rho})}{Z_{\mathrm{g}}^{\mathrm{TE}, \mathrm{TE}}(k_{\rho}) + Z_{\mathrm{d}}^{\mathrm{TE}, \mathrm{TE}}(k_{\rho})}. \tag{11}
\]

where \(k_{\rho}\) is the wavenumber transverse to the line source in the plane of the grid. For a grounded slab having thickness \(h\) and characterized by \(\mu_2, \varepsilon_2\), the dielectric impedances \(Z_{\mathrm{d}}\) for TM\(_z\) and TE\(_z\) waves are [26]
\[
Z_{\mathrm{d}}^{\mathrm{TM}}(k_{\rho}) = j \omega \mu_2 \frac{\tanh(p_2h)}{p_2} \left( 1 - \frac{k_{\rho}^2}{k_{\rho}^2} \right) \tag{12}
\]
\[
Z_{\mathrm{d}}^{\mathrm{TE}}(k_{\rho}) = j \omega \mu_2 \frac{\tanh(p_2h)}{p_2} \tag{13}
\]

where \(p_2^2 = k_{\rho}^2 - k_{\rho}^2\). Grid impedances \(Z_{\mathrm{g}}\) for various metallizations (strips, patches, and Jerusalem crosses) are provided in the appendix (assuming for simplicity nonmagnetic media).

The scattered potential is obtained by enforcing the boundary condition at \(z = 0\) as
\[
\mathbf{\hat{z}} \times \mathbf{H} = \mathbf{z}/Z_{\mathrm{g}} \tag{14}
\]

where \(Z_{\mathrm{g}}\) is given by (11), leading to the coefficients
\[
R_t = \frac{p_1 - j \omega \varepsilon_1 Z_{\mathrm{g}}^{\mathrm{TM}}}{p_1 + j \omega \varepsilon_1 Z_{\mathrm{g}}^{\mathrm{TM}}} \tag{15}
\]

2) **Method II: Two-Sided Impedance Surface in Layered Medium**: In this method, we again assume that the periodic metallization is replaced by an homogenized grid with impedance \(Z_{\mathrm{g}}\) (the same \(Z_{\mathrm{g}}\) as in Method I), which now lies on top of a grounded slab, as shown in Fig. 3.

In this case, the boundary conditions are
\[
\mathbf{\hat{z}} \times (\mathbf{H}^+ - \mathbf{H}^-) = \frac{1}{Z_{\mathrm{g}}} \mathbf{E} \tag{16}
\]
\[
\mathbf{\hat{z}} \times (\mathbf{E}^+ - \mathbf{E}^-) = 0
\]

at \(z = 0\), and \(\mathbf{z} \times \mathbf{E} = \mathbf{0}\) at \(z = -h\) (at the ground plane). Thus, rather than using the slab impedance \(Z_{\mathrm{d}}\), we solve the boundary-value problem of an impedance surface in a layered medium. This leads to the spectral coefficients
\[
R_t(k_{\rho}) = \frac{\tanh(p_2h)}{p_2} \left( \frac{p_1 - k_{\rho}^2}{p_1 + k_{\rho}^2} \right) \tag{17}
\]
\[
R_n(k_{\rho}) = \frac{p_1 - p_2 \tanh(p_2h) \frac{1}{p_1}}{p_1 + p_2 \tanh(p_2h) \frac{1}{p_1}} \tag{18}
\]
For observation points in the half-space $z > 0$ as considered above, $R_{\text{method I}} = R_{\text{method II}}$.

### III. HOMOGENIZED MODEL AND ASM

Some general considerations on the accuracy of the above-described homogenized model and a short description of the rigorous full-wave approach (based on the ASM) adopted here for comparison and validation purposes are provided in this section.

#### A. Accuracy of the Homogenized Model for Aperiodic Line Sources

Qualitative insight into the validity of homogenized representations for the description of the field excited by an aperiodic line source can be gained by simple considerations of dimensional ratios between the characteristic lengths involved in the problem, i.e., the spatial periods $p_x$, $p_y$, the free-space wavelength $\lambda_0$, and the distances of the source plane ($z_0$) and the observation plane ($z$) from the periodic grid.

For a homogenized representation of a periodic grid in terms of a surface impedance to be valid it is required that: 1) the incident field produced by the line source, which is independent of the $y$-coordinate, varies little over one spatial period $p_x$, in the $x$-direction and 2) the evanescent field scattered by the screen is negligible at the observation point.

Considering the plane-wave spectrum representation (6) of the incident field, evaluated on the plane of a densely periodic grid $z = 0$, condition 1) above is certainly satisfied by the plane waves in the visible part of the spectrum ($|k_{z}\| \leq \lambda_1$), since $p_x \ll \lambda_1$ implies $k_1 \ll 2\pi/p_x$. On the other hand, from the term $\exp(-p_1z_0)$ in (6), we see that the spectrum in the evanescent region ($k_{z} > k_{1}$) decays (at least) exponentially as a function of $k_{z}$; hence, it can be neglected for $p_1z_0 > M$, where $M$ is a suitable constant, i.e., for $k_{z} > k_{x}\| = \sqrt{k_1^2 + (M/z_0)^2}$. By requiring $k_{z} = k_{x}\| \ll 2\pi$ and taking into account that $k_{z} \ll \lambda_1$, a lower bound for $z_0$ is obtained in the form $z_0 > M/p_x/(2\pi)$. From our numerical experiments, we have found that a sound choice for $M$ is $M = 2\pi$, resulting in the lower bound $z_0 > p_x$.

Assuming that the latter condition is satisfied, for each plane wave in the spectrum of the incident field, the field scattered by the grid admits a Floquet representation in which all the higher-order space harmonics are evanescent. Condition 2) above can then be enforced by requiring that the first higher order space harmonic (i.e., the one with the smallest attenuation constant in the $z$-direction) has a negligible amplitude at the observation point. For a densely periodic screen and for each plane wave in the spectrum of the incident field, the scattered field has only one propagating space harmonic; furthermore, the attenuation constant of the first higher order evanescent space harmonic can be approximated as $\alpha_z \simeq 2\pi/p_z$, where $p_z = \max(p_x, p_y)$. Condition 2) requires that $\alpha_z z > M'$, where again $M'$ is a suitable constant; this sets a lower bound for $z$ in the form $z > M' p_z/(2\pi)$. The constant $M'$ depends on the specific geometry of the periodic screen inside the unit cell, since this affects the excitation amplitude of the higher order space harmonics. It is very difficult to determine analytically such a dependence; however, from our numerical experiments, it turns out that, by letting $M' = 2\pi$, the homogenized model gives good results; this results in the lower bound $z > p$. It is to be noted that, while generally valid, this criterion may result in an unduly restrictive condition for specific grid geometries; the case of metal strips parallel to the line source is one example, as illustrated in Section IV.

In conclusion, we stress that the above-discussed lower bounds for $z_0$ and $z$ provide only sufficient conditions for the accuracy of homogenized models in representing the exact, microscopic field. They have been obtained empirically from the analysis of a number of specific grids and their validity will be illustrated in Section IV on selected structures. As a final remark, we note that the average of the microscopic field over one period (i.e., by definition, the macroscopic field) is usually very well represented by the homogenized models considered here, hence the agreement between homogenized and averaged exact fields can be very good also when the above-discussed lower bounds are not satisfied.

#### B. Full-Wave ASM Method

A full-wave ASM method has been used to verify the presented homogenized Green’s function models. The ASM method is based on the representation of the aperiodic line source as an integral superposition of auxiliary Floquet-periodic sources, obtained by periodically replicating (see Fig. 2) the aperiodic source along the $x$-direction with phase shift $k_x p_x \in (-\pi, +\pi)$, where $p_x$ is the spatial period along the $x$-direction. By linearity, once the electric field excited by the auxiliary sources has been determined, the electric field solution of the original problem can be obtained through a similar spectral integral superposition.

The determination of the auxiliary Floquet-periodic fields has been performed here by discretizing the relevant electric-field integral equation (EFIE) with the method of moments (MoM) in the spatial domain. A fully general code for the analysis of planar structures periodic along two directions (2-D periodic) with arbitrary metallizations within the unit cell has been used, employing Rao–Wilton–Glisson basis functions and adopting a Galerkin testing scheme [39]. A simpler MoM code for the analysis of a metal strip grating (1-D periodic) with strips parallel to the source has also been employed, with entire-domain basis functions defined on the strip cross section [35]. In both codes, a crucial aspect is the use of accelerated periodic Green’s functions [40] to reduce the computation time and, hence, allowing for a calculation of the involved spectral integrals in a reasonable time. More details on the ASM approach for the analysis of a line source in the presence of a 2-D periodic structure can be found in [41].

### IV. NUMERICAL RESULTS

Here, we compare results obtained using the homogenized Green’s functions and the ASM for a variety of grid geometries. We remark that the computation times required by the ASM are typically from two to three orders of magnitude larger than those required by the homogenized models. The involved physical and geometrical parameters are varied in order to illustrate their effect on the accuracy of the relevant homogenized representations.

Figs. 4–8 show results for a line source over metal strips oriented parallel to the source, as depicted in the inserts of the fig-
The dielectric permittivity is $\varepsilon_r = 10.2$. Results obtained with ASM-MoM methods with both 2-D and 1-D periodicity (see Section III-B) are reported in order to validate the 2-D approach for the cases when the homogenized model of the periodic structure is accurate and when it fails to reproduce the exact field.

In Fig. 4, the period is $p_x = 2\, \text{mm}$ ($p_x/\lambda_0 = 0.1$ at 15 GHz), the line source is located at $(x_0, z_0) = (0, 0.5)\, \text{mm}$ ($z_0/\lambda_0 = 0.025$ at 15 GHz), and the observation points are located at $(10, z)\, \text{mm}$, where results for $z = 1\, \text{mm}$, $z = 0.8\, \text{mm}$, and $z = 0.1\, \text{mm}$ correspond to $z/\lambda_0 = 0.05$, $z/\lambda_0 = 0.04$, and $z/\lambda_0 = 0.005$ at 15 GHz, respectively. Note that the line source is located right above the center of a metal strip. This figure shows that the homogenized model is very accurate in a wide frequency range and also for observation points very close to the periodic structure, i.e., even for $z_0$ and $z$ smaller than $p_x$ (see Section III-A). This can be expected taking into account that the metal strips are relatively wide, so that the effect of the gaps is small, and the grating is acting effectively as a homogeneous good conductor. This in turn implies that higher order evanescent space harmonics are only weakly excited, thus producing a very good agreement between homogenized and full-wave results.

In order to explore the limits of validity of the homogenized representation, we consider now the behavior of the field as a function of the horizontal abscissa $x$ when the source is located at $z_0 = 3\, \text{mm}$, i.e., above the lower bound $z_0 > p_x$ given in Section III-A ($z_0/p_x = 1.5$), for the same distances between grid and observation point as in Fig. 4. In Fig. 5, a very good agreement between homogenized and full-wave results can still be observed. By reducing the ratio $w/p_x$, the excitation of the higher order space harmonics is expected to increase, thus reducing the accuracy of the homogenized representation. To investigate the effect of varying $w/p_x$, the excitation of the higher order space harmonics is expected to increase, thus reducing the accuracy of the homogenized representation. To investigate the effect of varying $w/p_x$, in Figs. 6 and 7, a geometry as in Fig. 5 is considered except that $w = 1\, \text{mm}$, thus reducing $w/p_x$ from 0.9 to 0.5. The observation point in Fig. 6 is $z = 1\, \text{mm}$ ($z/\lambda_0 = 0.05$) and $z = 0.8\, \text{mm}$ ($z/\lambda_0 = 0.04$), whereas in Fig. 7 it is $z = 0.1\, \text{mm}$ ($z/\lambda_0 = 0.005$). As expected, while excellent agreement is found for $z = 1\, \text{mm}$ and $z = 0.8\, \text{mm}$, extremely close to the metallization ($z/\lambda_0 = 0.005$), a strong disagreement is observed. In this case, the evanescent field scattered by the grid is not negligible and is responsible for the oscillation of the field. As observed in Section III-A, the average
value of the field over one spatial period is anyway well represented by the homogenized model; this is also clearly visible in Fig. 7, taking into account that the field is independent of the $y$-coordinate and the structure is periodic along the $x$-axis.

Finally, Fig. 8 illustrates the effect of increasing the normalized period $p_x/\lambda_0$ by keeping $w/p_x$ and the other parameters fixed; in particular, now $p_x = 5$ mm, so that $p_x/\lambda_0 = 0.25$ at 15 GHz, and $w = 2.5$ mm, so that again $w/p_x = 0.5$. For larger values of the normalized period, the accuracy of the homogenized model decreases; in this case, a discrepancy with respect to the full-wave results can be appreciated already at $z = 0.8$ mm.

The results shown so far for a line source parallel to the strip grating have illustrated the validity and limits of the homogenized model and have validated the accuracy of the ASM-MoM code with 2-D periodicity against an independent ASM-MoM approach with 1-D periodicity. In the following, periodic structures will be considered for which a 1-D periodic analysis as in [35] is not possible.

In Figs. 9 and 10, the strips are oriented perpendicular to the line source (see also Fig. 20). As in Figs. 5–7, $p_y = 2$ mm, $h = 1$ mm, and $\varepsilon_r = 10.2$, $z_0 = 3$ mm, and $f = 15$ GHz. In Fig. 9 the strip width is $w = 1.8$ mm (as in Fig. 5), whereas in Fig. 10 $w = 1$ mm (as in Figs. 6 and 7). For source or field points sufficiently far above the metallization, i.e., $z > p_y$, the homogenized Greens’s functions are accurate. However, when $z < p_y$, their accuracy breaks down, as can be observed both in Figs. 9 ($w/p_y = 0.9$) and 10 ($w/p_y = 0.5$) when $z = 0.1$ mm ($z/\lambda_0 = 0.005$). Furthermore, by reducing the spatial period to $p_y = 0.2$ mm (i.e., $p_y/\lambda_0 = 0.01$ at 15 GHz) by keeping $w/p_y$ and the other parameters fixed, a good agreement can be restored between homogenized and full-wave results, as can be observed in Fig. 11.

In Figs. 12–15, a square array of metal patches is considered, with $p = p_x = p_y = 2$ mm, $h = 1$ mm, $\varepsilon_r = 10.2$, and $z_0 = 3$ mm (see also Fig. 21). In Figs. 12 and 13, $g = 0.2$ mm and $f = 15$ GHz. In Fig. 12, the cases $z = 8$ mm and $z = 2$ mm, for which $z > p$, show a good agreement between homogenized and full-wave results. In Fig. 13, a disagreement is observed at $z = 0.5$ mm. By plotting the field at $x = 10$ mm as a function of frequency for the values of $z$ considered in Fig. 12, we see in Fig. 14 that a good agreement is maintained over the entire shown frequency range. Incidentally, a typical resonant behavior can be observed, with the field amplitude exhibiting a maximum close to the frequency at which the periodic interface behaves as a perfect magnetic conductor for normally incident plane waves.

Fig. 15 shows results for a patch array with $g = 1$ mm at 15 GHz, for observation points ranging from $z = 8$ mm ($z/\lambda_0 = 0.4$) to $z = 0.5$ mm ($z/\lambda_0 = 0.025$). For $z = 1$ mm
Fig. 12. Near electric field (in magnitude) excited by a line source above a grounded dielectric slab covered with an array of square metal patches, as a function of the abscissa $x$ at $f = 15$ GHz: comparison between the homogenized model (solid gray line) and ASM-MoM results with 2-D periodicity (black crosses). Parameters: $h = 1$ mm; $\varepsilon_r = 10.2$; $p = p_x = p_y = 2$ mm; $g = 0.2$ mm; $\varepsilon_0 = 3$ mm.

Fig. 13. Same as in Fig. 12 at $z = 1$ mm and $z = 0.5$ mm. ASM-MoM results with 2-D periodicity are represented with black crosses ($z = 1$ mm) or diamonds ($z = 0.5$ mm).

Fig. 14. Near electric field (in magnitude) excited by a line source above a grounded dielectric slab covered with an array of square metal patches, as a function of frequency at $x = 10$ mm: comparison between the homogenized model (solid gray line) and ASM-MoM results with 2-D periodicity (black crosses). Parameters: the same as in Fig. 12.

Fig. 15. Same as in Figs. 12 and 13, except for $g = 1$ mm.

Fig. 16. Same as in Fig. 13, except for $p = 0.5$ mm and $g = 0.25$ mm.

Fig. 17. Near electric field (in magnitude) excited by a line source above a grounded dielectric slab covered with an array of metal Jerusalem crosses, as a function of the abscissa $x$ at $f = 10$ GHz: comparison between the homogenized model (solid gray line) and ASM-MoM results with 2-D periodicity (black crosses). Parameters: $g = 0.1$ mm; $d = 2.8$ mm; $t = w = 0.2$ mm; $p = p_x = p_y = 4$ mm; $h = 6$ mm; $\varepsilon_r = 2.7$; $\varepsilon_0 = 10$ mm.

Finally, Figs. 17 and 18 show results for a Jerusalem-cross structure with $g = 0.1$ mm, $d = 2.8$ mm, $t = w = 0.2$ mm, $p = p_x = p_y = 4$ mm, $h = 6$ mm, and $\varepsilon_r = 2.7$ (see also Fig. 22). In Fig. 17, $f = 10$ GHz and $\varepsilon_0 = 10$ mm: agreement between the two methods is excellent for the shown values of $(z/\lambda_0 = 0.05)$, the agreement between the homogenized approach and the ASM begins to deteriorate. Again, by reducing the normalized period, the agreement improves, as illustrated in Fig. 16 for the case $p = 0.5$ mm ($p/\lambda_0 = 0.025$) and $g = 0.25$ mm.
\[ z. \] In Fig. 18, the source distance from the screen is reduced to \( z_0 = 3 \) mm; the case \( z = 2 \) mm is considered for \( f = 10 \) GHz and \( f = 3 \) GHz. At the former frequency, comparison with Fig. 17 shows that, by reducing \( z_0 \), the accuracy of the homogenized model decreases. As expected, by reducing frequency to 3 GHz, hence reducing \( p/\lambda_0 \), the agreement with full-wave results is restored.

V. CONCLUSION

A homogenized line-source Green’s function model that treats planar, densely periodic metallization patterns as quasi-dynamic homogenized impedance surfaces has been described. Two methods were presented to obtain the homogenized Green’s functions in layered media. In the first method, the entire structure is modeled as a one-sided impedance surface, and, in the second method, the metallization is treated as a homogenized impedance surface embedded in a layered medium. In both cases, the dielectric layers are treated in a fully dynamic manner. The resulting Green’s functions are relatively simple in form and very efficient to evaluate compared with full-wave methods. Extensive numerical results have been shown to demonstrate the accuracy of the presented homogenized analysis.

Although the accuracy of the homogenized Green’s functions varies depending on grid parameters and frequency (e.g., \( p_x/\lambda_0 \) and \( w/p_y \)), for a range of parameters likely to be encountered in applications, the homogenized Green’s functions are very accurate, even for source and observation points very close to the grid, and only begin to lose accuracy for \( z, z_0 < 0.05\lambda_0 \).

APPENDIX

The grid impedance \( Z_{g} \) is a uniform surface impedance that arises from a planar homogenization of the grid metallization \([25]–[29]\) and obviously varies with the type of grid. Several metallization patterns are considered in the following: parallel strips, square patches, and the Jerusalem cross. In each case, the line source is assumed to lie parallel to the \( y \)-axis, and we assume that the grid is in a nonmagnetic environment (\( \mu_1 = \mu_2 = \mu_0 \)) with \( \varepsilon_1 = \varepsilon_0 \) and \( \varepsilon_2 = \varepsilon_0 \varepsilon_r \).

For strips as in Fig. 19 (line source parallel to the strips) \([26]\)

\[
Z_{g}^{TE_{\|\text{strips}}} = j \frac{\eta_{\text{eff}}}{2} \frac{\alpha}{\alpha}
\]
\[
Z_{g}^{TM_{\|\text{strips}}} = j \frac{\eta_{\text{eff}}}{2} \alpha \left[ 1 - \frac{1}{2} \left( \frac{k_x}{k_{\text{eff}}} \right)^2 \right]
\]

where

\[
\alpha = \frac{k_{\text{eff}} p_x}{\pi} \ln \left( \frac{\pi w}{2p} \right)
\]

and \( \eta_{\text{eff}} = \sqrt{\mu_0/\varepsilon_0 \varepsilon_{\text{eff}}} \), \( k_{\text{eff}} = k_0 \sqrt{\varepsilon_{\text{eff}}} \), and where \( \varepsilon_{\text{eff}} = (\varepsilon_r + 1)/2 \) approximately accounts for the strips not being in free space.

For strips as in Fig. 20 (line source perpendicular to the strips) \([26], [28]\)

\[
Z_{g}^{TE_{\perp\text{strips}}} = -j \frac{\eta_{\text{eff}}}{2\alpha} \left[ 1 - \left( \frac{k_x}{k_{\text{eff}}} \right)^2 \right]^{-1}
\]
\[
Z_{g}^{TM_{\perp\text{strips}}} = -j \frac{\eta_{\text{eff}}}{2\alpha}
\]

where

\[
\alpha = \frac{k_{\text{eff}} p_y}{\pi} \ln \left( \frac{\pi (p - w)}{2p} \right)
\]
For an array of patches as depicted in Fig. 21 with $p_x = p_y = p$ [28]

\[
Z_{\text{TE,patch}}^{\text{TM}} = -\frac{j \eta_{\text{eff}}}{2\alpha} \left[ 1 - \frac{1}{2} \left( \frac{k_x}{k_{\text{eff}}} \right)^2 \right]^{-1} (25)
\]

\[
Z_{g,\text{patch}}^{\text{TM}} = -\frac{j \eta_{\text{eff}}}{2\alpha} (26)
\]

where the grid parameter is

\[
\alpha = \frac{k_{\text{eff}} p}{\pi} \ln \left[ \csc \left( \frac{\pi y}{2p} \right) \right]. (27)
\]

(This is the same expression as for the perpendicular strip array, where $p - w = g$ is the spacing between strips, except that the factor of $1/2$ in the patch formula is absent for the strip case.)

For a Jerusalem cross as shown in Fig. 22 with $p_x = p_y = p$ [27], [29]

\[
Z_{g,\text{JC}}^{\text{TM}} = \frac{j \omega L_{g,\text{JC}}^{\text{TM}}}{2 \omega} + \frac{1}{j \omega C_{g,\text{JC}}} (28)
\]

\[
Z_{g,\text{JC}}^{\text{TE}} = \frac{j \omega L_{g,\text{JC}}^{\text{TE}}}{2 \omega} + \frac{1}{j \omega C_{g,\text{JC}}} (29)
\]

where

\[
L_{\text{TM}} = \frac{\eta_{\text{eff}} \alpha}{2\omega} (30)
\]

\[
L_{\text{TE}} = \frac{\eta_{\text{eff}} \alpha}{2\omega} \left[ 1 - \left( \frac{k_x}{k_{\text{eff}}} \right)^2 \right]^{2} (31)
\]

\[
C_{g} = \frac{\varepsilon_{0} \varepsilon_{r} d}{\pi} \left[ \ln \left( \frac{\pi y}{2p} \right) + F \right] (32)
\]

with $\alpha$ given by (21) and

\[
F = \frac{\sqrt{1 - \left( \frac{y}{\lambda} \right)^2} u^2 + \left( \frac{\eta y (3u - 2)}{4 \lambda} \right)^2}{1 + \sqrt{1 - \left( \frac{y}{\lambda} \right)^2} (1 - u)^2} (33)
\]

\[
u = \cos^{2} \left( \frac{\pi y}{2d} \right). (34)
\]

References


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