PERTURBATION FORMULA FOR THE NATURAL FREQUENCIES OF AN OBJECT IN THE PRESENCE OF A LAYERED MEDIUM

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ABSTRACT

It is well known that the natural frequencies of an object are important distinguishing features which can be used in target detection and discrimination schemes. These natural frequencies are governed by the size, shape, and material composition of the object, as well as the environment in which the object resides. Since a given object of interest may reside in many different environments, it is of interest to relate the natural frequencies of an object when in free-space to the natural frequencies of the same object when immersed in a non-homogeneous environment. In this paper a perturbation formula is developed which relates the free-space natural frequencies of an object to those of the same object in the presence of a planarly layered medium. The perturbation formula is valid for intermediate spacing between the object and the nearest planar interface. Numerical results are shown for the natural frequencies of a wire in the presence of a layered medium.

1. INTRODUCTION

Determination of the natural frequencies of an object is important in target detection and discrimination methods, scattering analyses, and in other applications. The natural frequencies of an object are of paramount importance in the singularity expansion method (SEM) (Baum, 1971), and are generally recognized as aiding in the physical interpretation of electromagnetic interaction data. While traditionally natural frequencies have been computed for finite-sized objects in free-space, recently there has been some attention devoted to the

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determination of the natural frequencies of an object in the presence of a layered medium (Rothwell and Cloud, 1996), (Vitebskiy and Carin, 1996). Such efforts have been directed towards accounting for realistic environments such as an air-sea or air-ground interface in target detection and identification schemes.

Early in the development of SEM it was shown that for an integral equation (IE) treatment of finite-sized objects in free-space, the operator inverse to the integral operator is a meromorphic function in the complex frequency plane (Marin and Latham, 1972), leading to the occurrence of only pole-singularities in the current density response of the object (perhaps with the addition of an entire function term depending on the chosen time-space origin for the problem (Baum, 1992)). With this in mind, some early work on the natural frequencies of thin wire scatterers in free-space was described in (Tesche, 1973). A short time later the natural system frequencies of coupled wires were studied (Umashankar et al., 1975), (Shumpert and Galloway, 1978). It was found that the natural system frequencies of the two-wire configuration exhibited some interesting characteristics as wire separation was varied. In particular, for certain configurations the natural frequencies of two identical coupled wires tended to spiral about the natural frequency of the isolated wire as spacing between the wires was varied over some intermediate distance (Ross et al., 1994). As separation was further increased, the system resonances moved off towards the origin in the complex frequency plane, and other system modes moved in to take their place, again spiraling around the dominant isolated natural frequency.

Since the natural frequencies of a coupled system are rigorously obtained from a complicated (usually integral) system of equations, simple approximate formulas which describe the natural system resonance behavior as a function of body separation are of interest. For intermediate separations, perturbation formulas have been obtained which relate the natural system frequencies of two or more objects to the natural frequencies of the same objects when isolated. Two related classes of perturbation solution have been obtained, both based upon the exact integral-operator description of the coupled system. The first method yields a quasi-analytic formula for the system frequencies of an object and a mirror object, separated by some intermediate distance. The resulting formula involves a numerically computed coefficient which only depends upon the isolated object's characteristics, multiplied by an exponential term which is a function of the separation between the objects (Baum et al., 1989). The second method is more numerical in nature, yet represents a considerable simplification of the exact IEs and is applicable to a more general system of coupled bodies (Chuang and Nyquist, 1984). That formulation was subsequently applied to a variety of coupled objects (Ross et al., 1994), (Yuan and Nyquist, 1990), (Hanson and Nyquist, 1992). For the case of large separation between coupled objects, the system frequencies tend towards the origin in the complex frequency plane. An asymptotic formulation for this situation was described in (Hanson and Baum, 1997).

Although the previously described perturbation formulations have been developed for the natural frequencies of two or more coupled objects, the described spiraling behavior is not limited to coupling between finite-sized objects in homogeneous space. For instance, in (Rothwell and Cloud, 1996) it was observed that the lowest-order natural frequency of a wire ring over a lossy dielectric half-space exhibited a similar behavior, as spacing between the wire and material interface was varied. Similar findings were reported in (Vitebskiy and Carin, 1996) for a straight wire embedded in a lossy ground in the vicinity of the air-ground interface, and early work in (Riggs and Shumpert, 1979) describes results for a wire above a lossy ground. In this paper we develop a perturbation formula for the natural frequencies of an object over multi-layered media. This perturbation formula is based on an exact integral equation formulation. Subsequent approximations are then made to yield a useful formula which relates the natural frequencies of an object over a multi-layered medium to the natural frequencies of an object when in free-space. The perturbation formula is applied to several configurations of wires in the vicinity of material interfaces, and a discussion of the formula's applicability is provided.

It should be noted that for a finite-sized object embedded in a laterally infinite layered environment, the operator inverse to the integral operator is not a meromorphic function in the complex frequency plane (Hanson, 1997). Branch-point singularities also exist, which are associated with the propagation of surface waves in the layered medium. Although an SEM expansion for the current density response then includes branch-cut as well as pole singularity terms, the natural frequencies of such objects are still very important, and associated pole singularities may be expected to dominant the response for a wide variety of environments.

2. INTEGRAL EQUATION FORMULATION

Consider an object in the presence of a planarly layered medium, as depicted in Fig. 1. For a specified impressed field, an integral equation (IE) can be formed which leads to the determination of the current induced on/in the object. For generality, the object will be considered to either have a perfectly conducting surface, leading to a surface IE, or be composed of a lossy dielectric $(\epsilon = \epsilon_2)$, leading most simply to a volume IE. In either case, an electric field integral equation (EFIE) can be formed as

$$\left\langle \vec{\vec{Z}}(\vec{r}\,|\,\vec{r}\,', s) \;;\; \vec{J}(\vec{r}\,', s) \right\rangle = \vec{E}^{(inc)}(\vec{r}, s) \qquad (1)$$

where the bracket notation indicates a real inner product over common spatial coordinates. For perfectly conducting objects the surface IE is enforced over the surface of the body, whereas for dielectric objects the volume IE is enforced over the volume of the body.

The kernel for either IE can be written as

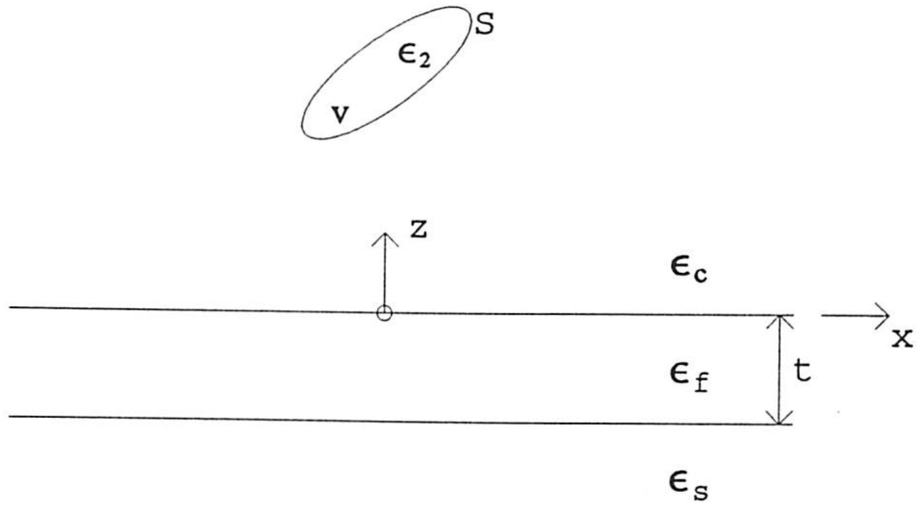


FIGURE 1. An object in the vicinity of a layered medium.

$$\overleftrightarrow{Z}(\vec{r}|\vec{r}\,',s) = s\mu_0 \overleftrightarrow{1}_{\binom{S}{V}}(\vec{r}) \cdot [\overleftrightarrow{G}^h(\vec{r}|\vec{r}\,',s) + \overleftrightarrow{G}^s(\vec{r}|\vec{r}\,',s)] \cdot \overleftrightarrow{1}_{\binom{S}{V}}(\vec{r}\,')$$
$$= \overleftrightarrow{Z}^h(\vec{r}|\vec{r}\,',s) + \overleftrightarrow{Z}^s(\vec{r}|\vec{r}\,',s) \tag{2}$$

where $\overleftrightarrow{Z}^h$ is the homogeneous space kernel (principal part of $\overleftrightarrow{Z}$), and $\overleftrightarrow{Z}^s$ is the scattered kernel which accounts for the material layering. In (2), $\overleftrightarrow{1}_{\binom{S}{V}}$ distinguishes between surface and volume formulations, where for the surface IE

$$\overleftrightarrow{1}_S(\vec{r}) = \overleftrightarrow{1}_t(\vec{r}) = \overleftrightarrow{1} - \vec{1}_S(\vec{r})\,\vec{1}_S(\vec{r}) \tag{3}$$

is the transverse dyadic at $\vec{r}$ on S, where $\vec{1}_S(\vec{r})$ is the unit normal to S at $\vec{r}$, with $\overleftrightarrow{1} = \vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z$ being the identity dyadic. The transverse dyadic is used to enforce the tangential boundary condition for the electric field at the surface of a perfectly conducting object. For the lossy dielectric object, $\overleftrightarrow{1}_V(\vec{r}) = \overleftrightarrow{1}$. The Green's functions are defined by

$$\overleftrightarrow{G}^{h}(\vec{r}|\vec{r}\,',s) = PV[\overleftrightarrow{1} - \gamma^{-2}\nabla\nabla]\cdot\overleftrightarrow{g}^{h}(\vec{r}|\vec{r}\,',s) + \gamma^{-2}\overleftrightarrow{L}(\vec{r})\,\delta(\vec{r}-\vec{r}\,') + \gamma^{-2}\left(\frac{\epsilon_2}{\epsilon_c}-1\right)^{-1}\overleftrightarrow{1}\,\delta(\vec{r}-\vec{r}\,') \tag{4}$$

$$\overleftrightarrow{G}^{s}(\vec{r}|\vec{r}\,',s) = [\overleftrightarrow{1} - \gamma^{-2}\nabla\nabla]\cdot\overleftrightarrow{g}^{s}(\vec{r}|\vec{r}\,',s) \tag{5}$$

where the last term in (4) only occurs for the volume IE. For a perfectly conducting object (surface IE), the last term in (4) is omitted. In (4), $\overleftrightarrow{L}$ is a depolarizing dyadic (Yaghjian, 1980), the contribution from which is removed by the transverse dyad for the surface IE, but remains for the volume IE. The PV notation indicates that the corresponding term be integrated in a principal value sense by removing from the integration a small patch (surface IE) or small volume (volume IE) centered at $\vec{r}=\vec{r}\,'$.

In (4),(5), the potential Green's terms are

$$\overleftrightarrow{g}^{h}(\vec{r}|\vec{r}\,',s) = \overleftrightarrow{1}\frac{e^{-\gamma R}}{4\pi R} = \iint_{-\infty}^{\infty}\frac{\overleftrightarrow{1}}{2(2\pi)^2 p}\,e^{-p(z-z')}\,e^{j\vec{\lambda}\cdot(\vec{r}-\vec{r}\,')}\,d^2\lambda \tag{6}$$

for a homogeneous space in either spatial or spectral form, and

$$\overleftrightarrow{g}^{s}(\vec{r}|\vec{r}\,',s) = \iint_{-\infty}^{\infty}\frac{\overleftrightarrow{F}(\lambda,s)}{2(2\pi)^2 p}\,e^{-p(z+z')}\,e^{j\vec{\lambda}\cdot(\vec{r}-\vec{r}\,')}\,d^2\lambda \tag{7}$$

for the scattered part. In the above, $\gamma = s\sqrt{\mu_0\epsilon_c}$, $R=|\vec{r}-\vec{r}\,'|$, $\vec{\lambda}=\vec{1}_x k_x+\vec{1}_y k_y$, $d^2\lambda = dk_x dk_y$, $\lambda^2=k_x^2+k_y^2$, and $p=\sqrt{\lambda^2+\gamma^2}$. The wavenumber parameter $p(\lambda)$ is multivalued, necessitating the definition of an appropriate branch cut in the complex λ-plane. Unless otherwise specified, we'll consider the permittivity parameter to be generally complex, i.e., $\epsilon = \epsilon^{re}+\sigma/s$, with (ϵ^{re},σ) the real-valued permittivity and conductivity, respectively.

In the scattered Greens' function, $\overleftrightarrow{F}(\lambda, s)$ is an amplitude dyadic which is obtained by matching boundary conditions at the layering interfaces. For the configuration depicted in Fig. 1, this term can be expressed as

$$\vec{F}(\lambda, s) = (\vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y)R_t(\lambda, s) + \vec{1}_z\vec{1}_z R_n(\lambda, s) + \vec{1}_z(\vec{1}_x jk_x + \vec{1}_y jk_y)R_c(\lambda, s) \quad (8)$$

where R_t, R_n, R_c are given in (Bagby and Nyquist, 1987) for the tri-layered environment shown. Note that the presence of additional layering below the object, i.e., more than three layers, only affects the coefficients R_t, R_n, R_c, and not the general form of (8).

Associated with (1) are natural mode solutions which exist in the absence of an excitation (Baum, 1971). These modes (natural frequencies and corresponding natural current distributions) satisfy the homogeneous form of (1),

$$\left\langle \vec{\vec{Z}}(\vec{r}|\vec{r}', s_\alpha) \; ; \vec{J}_\alpha(\vec{r}') \right\rangle = \vec{0} \quad (9)$$

where s_α is the natural frequency, and $\vec{J}_\alpha$ is the associated natural current distribution (surface or volume). Equation (9) can be cast as a scalar relation by dot multiplication on the left by the natural mode current, leading to

$$\left\langle \vec{J}_\alpha(\vec{r}) ; \vec{\vec{Z}}(\vec{r}|\vec{r}', s_\alpha) \; ; \vec{J}_\alpha(\vec{r}') \right\rangle = 0 \; . \quad (10)$$

It is convenient to define a local coordinate system centered at some appropriate point in V as shown in Fig. 2, such that $\vec{r} = \vec{r}_c + \vec{r}_1$.

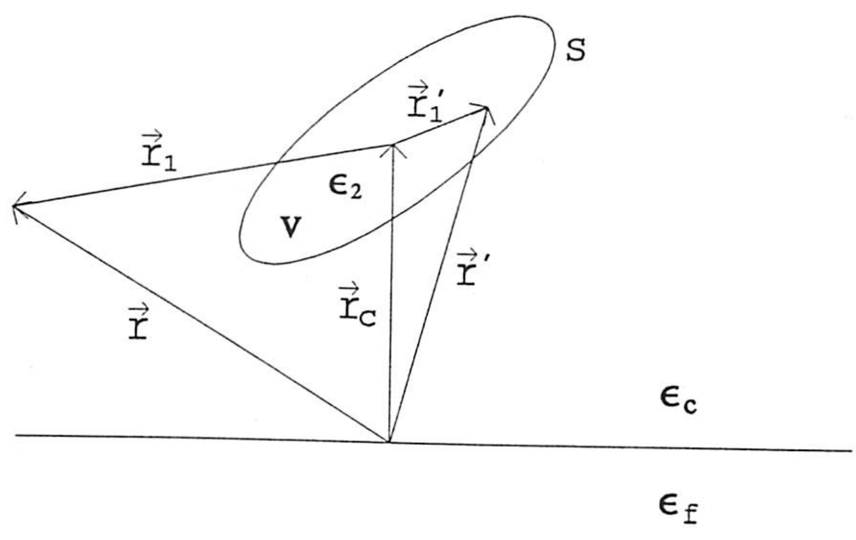


FIGURE 2. Local coordinate system used in development of perturbation formula.

For simplicity, assume $\vec{r}_c = \vec{1}_z b/2$. The integral relation (10) can be expressed in terms of the local coordinate system as

$$\left\langle \vec{J}_\alpha(\vec{r}_1) ; \vec{\vec{Z}}(\vec{r}_1|\vec{r}_1', s_\alpha, b) \;\; ; \vec{J}_\alpha(\vec{r}_1') \right\rangle = 0 \; . \tag{11}$$

In (11), the spectral forms for the Green's functions have become

$$\vec{\vec{g}}^{\,h}(\vec{r}_1|\vec{r}_1', s) = \int\!\!\int_{-\infty}^{\infty} \frac{\vec{\vec{1}}}{2(2\pi)^2 p} e^{-p(z_1 - z_1')} e^{j\vec{\lambda}\cdot(\vec{r}_1 - \vec{r}_1')} d^2\lambda$$
$$\vec{\vec{g}}^{\,s}(\vec{r}_1|\vec{r}_1', s, b) = \int\!\!\int_{-\infty}^{\infty} \frac{\vec{\vec{F}}(\lambda, s)}{2(2\pi)^2 p} e^{-pb} e^{-p(z_1 + z_1')} e^{j\vec{\lambda}\cdot(\vec{r}_1 - \vec{r}_1')} d^2\lambda \tag{12}$$

with an associated change of coordinates in (4), (5).

3. PERTURBATION THEORY

In this section we will consider the case when the material layering has a sufficiently small effect on the object's natural frequency that it may be treated as a perturbation of the homogeneous space result. First, define the appropriate relationship for the situation $\epsilon_f = \epsilon_s = \epsilon_c$ (object in homogeneous space), such that $\vec{\vec{Z}}^{\,s} = \vec{\vec{0}}$, and (11) becomes

$$\left\langle \vec{J}_\alpha^{\,h}(\vec{r}_1) ; \vec{\vec{Z}}^{\,h}(\vec{r}_1|\vec{r}_1', s_\alpha^h) \;\; ; \vec{J}_\alpha^{\,h}(\vec{r}_1') \right\rangle = 0 \; . \tag{13}$$

For later purposes, define the special case of the homogeneous space being free-space as

$$\left\langle \vec{J}_\alpha^{\,0}(\vec{r}_1) ; \vec{\vec{Z}}^{\,0}(\vec{r}_1|\vec{r}_1', s_\alpha^0) \;\; ; \vec{J}_\alpha^{\,0}(\vec{r}_1') \right\rangle = 0 \; . \tag{14}$$

Returning to (11), under certain conditions, $\|\vec{\vec{g}}^{\,s}\| < \|\vec{\vec{g}}^{\,h}\|$ (for some appropriately defined norm), and $\vec{\vec{Z}}^{\,s}$ can be treated as a small perturbation of $\vec{\vec{Z}}^{\,h}$ in (2). One such situation would be 'b' large (large separation between the object and the nearest material interface), but this is not necessary if the material parameters of the layers differ only slightly from the material parameters of the half-space in which the object resides. If we can assume $s_\alpha \approx s_\alpha^h + \Delta s_\alpha$, where s_α^h is the α th natural frequency of the object when in a homogeneous space characterized by (μ_0, ϵ_c) and Δs_α represents a small perturbation, then we may expect that the natural mode currents may be expressed as $\vec{J}_\alpha \approx \vec{J}_\alpha^{\,h} + \Delta \vec{J}_\alpha$. With the above assumption, expand the kernel in a Taylor's series about s_α^h

$$\vec{\vec{Z}}(\vec{r}_1|\vec{r}_1',s_\alpha) \approx \vec{\vec{Z}}(\vec{r}_1|\vec{r}_1',s_\alpha^h) + \frac{\partial}{\partial s}\vec{\vec{Z}}(\vec{r}_1|\vec{r}_1',s_\alpha^h)\,\Delta s_\alpha + O(\Delta s_\alpha^2) \tag{15}$$

where $O(\Delta s_\alpha^2)$ indicates terms which are at least quadratically small. It is understood in the second term that $s=s_\alpha^h$ is substituted after the frequency derivative is evaluated. Insertion of the two-term approximation for $(\vec{J}_\alpha, s_\alpha)$ into (11) and retaining the first non-vanishing term leads to the perturbation equation

$$\left\langle \vec{J}_\alpha^h(\vec{r}_1)\,;\,\frac{\partial}{\partial s}\vec{\vec{Z}}^h(\vec{r}_1|\vec{r}_1',s_\alpha^h)\,\Delta s_\alpha + \vec{\vec{Z}}^s(\vec{r}_1|\vec{r}_1',s_\alpha^h,b)\,;\,\vec{J}_\alpha^h(\vec{r}_1')\right\rangle = 0 \tag{16}$$

The above can be solved for the first-order perturbation in natural frequency due to the planar layering as

$$\Delta s_\alpha = -\frac{\left\langle \vec{J}_\alpha^h(\vec{r}_1)\,;\,\vec{\vec{Z}}^s(\vec{r}_1|\vec{r}_1',s_\alpha^h,b)\,;\,\vec{J}_\alpha^h(\vec{r}_1')\right\rangle}{\left\langle \vec{J}_\alpha^h(\vec{r}_1)\,;\,\frac{\partial}{\partial s}\vec{\vec{Z}}^h(\vec{r}_1|\vec{r}_1',s_\alpha^h)\,;\,\vec{J}_\alpha^h(\vec{r}_1')\right\rangle}\,. \tag{17}$$

While (17) may be computationally accurate under the assumption $\|\vec{\vec{g}}^s\| < \|\vec{\vec{g}}^h\|$, and represents an efficient formulation compared to the exact expression (10), in its present form it does not provide much physical insight into the effect of planar layering on the natural frequencies of an object. Accordingly, further restrictions may be stated to simplify (17). Let us assume that the separation 'b' between the object and the nearest interface is sufficiently large relative to dimensions of the object. For this case (12) can be reorganized as

$$\vec{\vec{g}}^s(\vec{r}_1|\vec{r}_1',s,b) = \int\!\!\int_{-\infty}^{\infty}\left\{\frac{\vec{\vec{F}}(\lambda,s)}{2(2\pi)^2 p}\,e^{-p(z_1+z_1')}\,e^{j\vec{\lambda}\cdot(\vec{r}_1-\vec{r}_1')}\right\}e^{-pb}\,d^2\lambda \tag{18}$$

where the term in brackets is slowly varying compared to e^{-pb} for 'b' large. The largest contribution to the integral will come from the point in the k_x-k_y plane where the phase of the rapidly varying exponential is stationary, i.e.,

$$\frac{\partial}{\partial k_x}(pb) = 0$$
$$\frac{\partial}{\partial k_y}(pb) = 0 \tag{19}$$

leading to $k_{x0} = k_{y0} = 0 \quad (\lambda = 0)$. Replacing the slowly varying part of the integrand with its value at $\lambda = 0$, results in

$$\vec{\vec{g}}^{\,s}(\vec{r}_1|\vec{r}_1^{\,\prime}, s, b) \approx \frac{\vec{\vec{F}}(0,s)}{2(2\pi)^2\gamma} e^{-\gamma(z_1+z_1^{\prime})} \iint_{-\infty}^{\infty} e^{-\sqrt{k_x^2+k_y^2+\gamma^2}\, b} \, d^2\lambda \, . \tag{20}$$

Converting the integral to polar form

$$I = \iint_{-\infty}^{\infty} e^{-\sqrt{k_x^2+k_y^2+\gamma^2}\, b} \, d^2\lambda = \int_0^{\infty}\int_0^{2\pi} e^{-\sqrt{\lambda^2+\gamma^2}\, b} \, \lambda \, d\theta \, d\lambda$$
$$= 2\pi \int_0^{\infty} e^{-\sqrt{\lambda^2+\gamma^2}\, b} \, \lambda \, d\lambda \tag{21}$$

the last integral can be easily evaluated in closed form to yield

$$I = 2\pi \left(\frac{\gamma}{b} - \frac{1}{b^2} \right) e^{-\gamma b} . \tag{22}$$

Retaining the leading term for 'b' large results in

$$\vec{\vec{g}}^{\,s}(\vec{r}_1|\vec{r}_1^{\,\prime}, s, b) \approx \vec{\vec{F}}(0,s) \frac{e^{-\gamma b}}{4\pi b} e^{-\gamma(z_1+z_1^{\prime})} \, . \tag{23}$$

With $\lambda = 0$, the amplitude coefficient for the scattered term becomes

$$\vec{\vec{F}}(0,s) = \left(\vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y\right) R_t(0,s) + \vec{1}_z\vec{1}_z R_n(0,s) \tag{24}$$

where the contribution from the coupling term associated with R_c in (8) has vanished.

The IE kernel for the scattered term in (17) then becomes

$$\vec{\vec{Z}}^{s}(\vec{r}_1|\vec{r}_1',s,b) = s\mu_0 \frac{e^{-\gamma b}}{4\pi b} \vec{\vec{1}}_{\binom{S}{V}} \cdot \left\{ \left[\vec{\vec{1}} - \gamma^{-1} \vec{1}_z \vec{1}_z \frac{\partial^2}{\partial z_1^2} \right] \cdot \vec{\vec{F}}(0,s)\, e^{-\gamma(z_1+z_1')} \right\} \cdot \vec{\vec{1}}_{\binom{S}{V}}$$

$$= s\mu_0 \frac{e^{-\gamma b}}{4\pi b} e^{-\gamma(z_1+z_1')} \vec{\vec{1}}_{\binom{S}{V}} \cdot \left[\left(\vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y \right) R_t(0,s) \right] \cdot \vec{\vec{1}}_{\binom{S}{V}} . \tag{25}$$

Defining

$$\vec{\vec{1}}_{z_t} \equiv \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y \tag{26}$$

leads to

$$\Delta s_\alpha = -s_\alpha^h \mu_0 \frac{e^{-\gamma_\alpha^h b}}{4\pi b} R_t(0,s_\alpha^h) \frac{\left\langle \vec{J}_\alpha^h(\vec{r}_1) \, ; e^{-\gamma_\alpha^h (z_1+z_1')} \vec{\vec{1}}_{z_t} ; \vec{J}_\alpha^h(\vec{r}_1') \right\rangle}{\left\langle \vec{J}_\alpha^h(\vec{r}_1) \, ; \frac{\partial}{\partial s} \vec{\vec{Z}}^h(\vec{r}_1|\vec{r}_1',s_\alpha^h) \, ; \vec{J}_\alpha^h(\vec{r}_1') \right\rangle} \tag{27}$$

where $\gamma_\alpha^h = \gamma_\alpha(s_\alpha^h) = s_\alpha^h \sqrt{\mu_0 \epsilon_c}$ is the propagation constant in the cover evaluated at the natural frequency of the object when in a homogeneous space. Note that the dyadic triple dot product in (25) becomes $\vec{\vec{1}}_{z_t}$ for both the surface and volume IE formulation. For the volume IE, this follows simply from the fact that $\vec{\vec{1}}_V(\vec{r}) = \vec{\vec{1}}$. For the surface IE, the knowledge that the natural mode currents are tangential reduces the triple dot product accordingly. Defining

$$v_\alpha^h(s_\alpha^h) \equiv \frac{\left\langle \vec{J}_\alpha^h(\vec{r}_1) \, ; e^{-\gamma_\alpha^h (z_1+z_1')} \vec{\vec{1}}_{z_t} ; \vec{J}_\alpha^h(\vec{r}_1') \right\rangle}{\left\langle \vec{J}_\alpha^h(\vec{r}_1) \, ; \frac{\partial}{\partial s} \vec{\vec{Z}}^h(\vec{r}_1|\vec{r}_1',s_\alpha^h) \, ; \vec{J}_\alpha^h(\vec{r}_1') \right\rangle} \tag{28}$$

which is exactly the same coefficient utilized in (Baum et al., 1989) (although there the homogeneous space was specifically free-space), the perturbation formula becomes

$$\Delta s_\alpha = -s_\alpha^h \mu_0 \frac{e^{-\gamma_\alpha^h b}}{4\pi b} R_t(0, s_\alpha^h)\, v_\alpha^h(s_\alpha^h) \ . \tag{29}$$

This formula is identical to that obtained in (Baum et al., 1989) (Eq. (34) of that paper), except for the multiplicative term R_t, and the fact that the perturbation term is evaluated at s_α^h rather than s_α^0. If $\epsilon_c = \epsilon_0,\ \epsilon_f = \epsilon_s = -j\infty$, implementing free-space over a perfectly conducting ground plane, $R_t = -1$, and (29) reduces to the result in (Baum et al., 1989) for the antisymmetric mode of two coupled objects in free-space, as expected. For the case of a perfect magnetic conductor, $R_t = 1$, and (29) reduces to the solution for the symmetric mode of two coupled objects in a homogeneous space.

4. SCALING RELATIONS

The perturbation formula (29) involves the natural frequencies s_α^0 of an object embedded in a homogeneous space characterized by (μ_0, ϵ_c). As detailed in (Umashankar and Wilton, 1974), (Giri and Tesche, 1981), (Baum, 1993), scaling relations exist which relate the natural modes of an object in a lossy, homogeneous environment to those of the same object when in free-space. The relevant scaling relationship for natural frequency is

$$s_\alpha^h = -\frac{\sigma_c}{2\epsilon_c^{re}} + \sqrt{\left(\frac{\sigma_c}{2\epsilon_c^{re}}\right)^2 + \frac{\epsilon_0}{\epsilon_c^{re}}\left(s_\alpha^0\right)^2} \tag{30}$$

where $(\sigma_c, \epsilon_c^{re})$ are real-valued parameters of the homogeneous space, which are written in terms of a single complex (effective) permittivity as $\epsilon_c = \epsilon_c^{re} + \dfrac{\sigma_c}{s}$. Note that in the case of a lossless homogeneous space, the scaling relationship becomes simply $s_\alpha^h = s_\alpha^0 / \sqrt{\epsilon_{cr}}$, with ϵ_{cr} being the relative permittivity, $\epsilon_{cr} = \epsilon_c^{re}/\epsilon_0$. The scaling relationship (30) comes from equality of the propagation constants, $\gamma_\alpha^h(s_\alpha^h) = \gamma_\alpha^0(s_\alpha^0)$. The natural modes scale as

$$\vec{J}_\alpha^h = \vec{J}_\alpha^0 \tag{31}$$

and the coefficient (28) scales as (Baum, 1993)

$$v_\alpha^h(s_\alpha^h) = \left(\frac{2\sigma_c + 2s_\alpha^h\epsilon_c^{re}}{\sigma_c + 2s_\alpha^h\epsilon_c^{re}}\right) v_\alpha^0(s_\alpha^0) \ . \tag{32}$$

With the above, (29) becomes

$$\Delta s_\alpha = -s_\alpha^h\left(\frac{2\sigma_c + 2s_\alpha^h\epsilon_c^{re}}{\sigma_c + 2s_\alpha^h\epsilon_c^{re}}\right)\mu_0 \frac{e^{-\gamma_\alpha^0 b}}{4\pi b} R_t(0, s_\alpha^h)\, v_\alpha^0(s_\alpha^0) \ . \tag{33}$$

Since

$$\Delta s_\alpha^0 \equiv s_\alpha^0 \mu_0 \frac{e^{-\gamma_\alpha^0 b}}{4\pi b} v_\alpha^0(s_\alpha^0) \tag{34}$$

is the perturbation for the antisymmetric mode of two coupled objects in free-space as obtained in (Baum et al., 1989), (33) can be written as

$$\Delta s_\alpha = -\frac{s_\alpha^h}{s_\alpha^0}\left(\frac{2\sigma_c + 2s_\alpha^h\epsilon_c^{re}}{\sigma_c + 2s_\alpha^h\epsilon_c^{re}}\right) R_t(0, s_\alpha^h)\, \Delta s_\alpha^0 \tag{35}$$

where s_α^h is given by (30). This formula reduces to the correct result for the special case of an object in free-space over a perfectly conducting ground plane $(R_t = -1, \sigma_c = 0, \epsilon_c^{re} = \epsilon_0, s_\alpha^h = s_\alpha^0)$, and gives no perturbation in the absence of planar layers $(\epsilon_f = \epsilon_s = \epsilon_c)$, for which $R_t = 0$. Equation (35) provides the perturbation of the natural frequency of an object embedded in a half-space characterized by (μ_0, ϵ_c) over a planarly layered medium in terms of the perturbation of the same object in free-space over a perfect ground plane, for which perturbation formulas have been derived. Equivalently, the perturbation may be directly computed from (29).

The reflection coefficient for the tri-layered environment depicted in Fig. 1 is (Bagby and Nyquist, 1987)

$$R_t = R_{fc} + \frac{T_{cf} R_{sf} T_{fc}}{1 - R_{cf} R_{sf} e^{-2p_f t}} e^{-2p_f t} \tag{36}$$

where $R_{\alpha\beta} = \frac{p_\beta - p_\alpha}{p_\beta + p_\alpha}$, $T_{\alpha\beta} = \frac{2p_\alpha}{N_{\beta\alpha}^2 (p_\alpha + p_\beta)}$, $p_\beta^2 = \lambda^2 + s^2\mu_0\epsilon_\beta$, and $N_{\alpha\beta}^2 = \frac{\epsilon_\alpha}{\epsilon_\beta}$. For the special case of a two half-spaces ($\epsilon_f = \epsilon_s$ or t=0), R_t

becomes

$$R_t = \frac{p_c - p_s}{p_c + p_s} \tag{37}$$

which for $\lambda=0$ is

$$R_t = \frac{\sqrt{\epsilon_c} - \sqrt{\epsilon_s}}{\sqrt{\epsilon_c} + \sqrt{\epsilon_s}} \tag{38}$$

a familiar interfacial reflection coefficient.

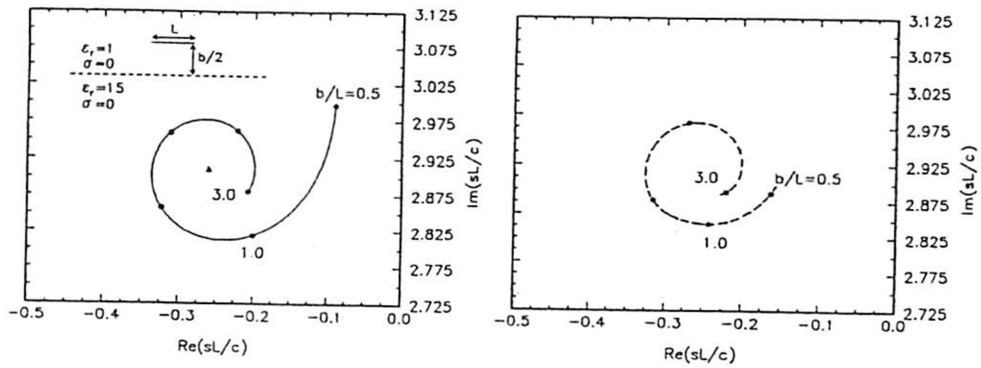
5. RESULTS AND DISCUSSION

As a check on the accuracy of perturbation formula (35), the example of a straight wire of length L in the vicinity of the interface between two differing media will be considered. Results are provided for a wire in air above a lossless and lossy half-space, and for a wire embedded in a lossy half-space near the interface with air. In all cases, numerical values of $\gamma^0_{1,1}$ and $v^0_{1,1}$ are obtained from tables in (Baum et al., 1989), leading to the coefficient (34). The subscript $\alpha=1,1$ indicates the lowest order mode in the first layer of the complex plane (Tesche, 1973), which is the dominant natural frequency. In the following all results correspond to $\alpha=1,1$, the perturbation of the dominant natural frequency of the isolated wire.

The first example considered is a wire in air $(\epsilon_{cr}=1, \sigma_c=0)$ over a dielectric half-space $(\epsilon_{sr}=15, \sigma_s=0)$, depicted in the insert of Fig. 3(a). The wire has length L and radius a, with L/a=200. Since the wire is in air, $s^h = s^0$, and the perturbation formula becomes simply $\Delta s_{1,1} = -R_t(0, s^0_{1,1})\,\Delta s^0_{1,1}$ with R_t computed from (38).

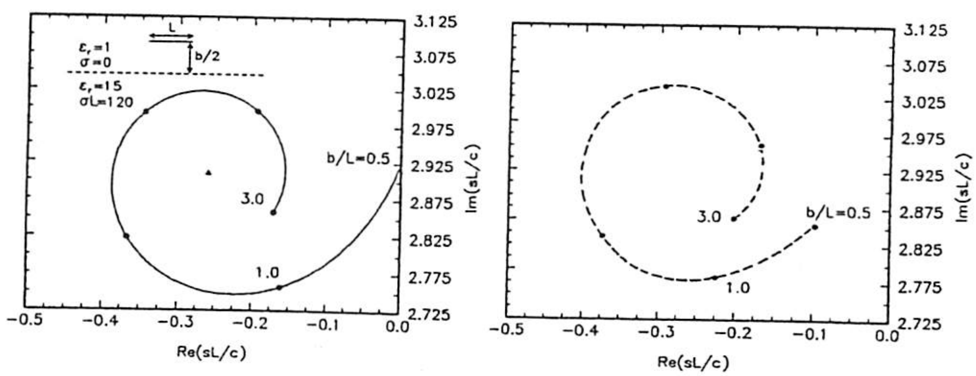
Results of the perturbation formula are shown in Fig. 3(a), where the separation parameter is varied from b/L=0.5 to 3.0. Results from an integral equation solution (Riggs and Shumpert, 1979) are shown in Fig. 3(b), where generally good agreement is found between the two methods for the range of b/L values examined.

The second example is depicted in the insert of Fig. 4(a), which shows a wire in air $(\epsilon_{cr}=1, \sigma_c=0)$ over a lossy dielectric half-space $(\epsilon_{sr}=15, \sigma_s L=120\,S)$ with L/a=200. The perturbation formula is the same as for the geometry considered in Fig. 3, although the substrate permittivity is now complex-valued in the coefficient R_t. Results of the perturbation formula are shown in Fig. 4(a), where the separation parameter is again varied from b/L=0.5 to 3.0. Results from an integral equation solution (Riggs and Shumpert, 1979) are shown in Fig. 4(b). Agreement between the two methods is again fairly good.



(a) (b)

FIGURE 3. (a) Results of perturbation formula (35) for the natural frequencies of a wire in air above a lossless dielectric half-space. Triangle denotes the natural frequency for a wire in free-space. (b) IE results (Riggs and Shumpert, 1979) for the natural frequencies of a wire in air above a lossless dielectric half-space.

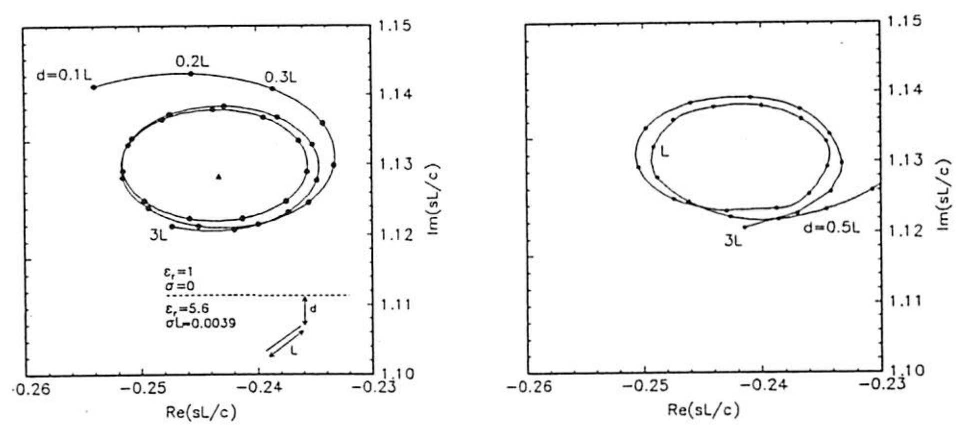


(a) (b)

FIGURE 4. (a) Results of perturbation formula (35) for the natural frequencies of a wire in air above a lossy dielectric half-space. Triangle denotes the natural frequency for a wire in free-space. (b) IE results (Riggs and Shumpert, 1979) for the natural frequencies of a wire in air above a lossy dielectric half-space.

Note that the value of conductivity taken in (Riggs and Shumpert, 1979) is very large, and so the results of Fig. 4 differ only slightly from those of a wire above a perfectly conducting ground plane. Comparing Fig.'s 3 and 4, it can be seen that the difference in results for a wire in air above a ground plane and above a lossless dielectric is governed by the coefficient $R_t(0, s_\alpha^h)$, which is -0.5896 for $(\epsilon_{sr}=15, \sigma_s=0)$. Thus, the radius of the spiral for the wire above the lossless dielectric is about one-half that of the wire above a ground plane.

The geometry for the last example is depicted in the insert of Fig. 5(a). For this situation, the wire is in a lossy ground near the interface with an air half-space. The permittivity of the ground is $\epsilon_c = (5.62 - j1.31)\epsilon_0$ from Fig. 3 of (Vitebskiy and Carin, 1996), leading to $(\epsilon_{cr}=5.62,$ $\sigma_c L = 0.0039\ S)$ at f=65.36 MHz for an L=0.825 m wire. The wire has radius 0.00556 m, resulting in L/a=148. The coefficient $v_{1,1}^0$ was taken from (Baum et al., 1989) for L/a=200, since values were not provided for smaller L/a ratios. The wire is inclined at a 45° angle to the planar interface, and separation between the end of the wire nearest to the interface and the interface is varied over the range specified in (Vitebskiy and Carin, 1996), i.e., from d=0.1L to 3L. Fig. 5(a) shows the perturbation formula results, while Fig. 5(b) shows the IE results from (Vitebskiy and Carin, 1996). Good agreement is found between the two methods except for small d/L values, which is expected.



(a) (b)

FIGURE 5. (a) Results of perturbation formula (35) for the natural frequencies of an inclined wire (45 degrees) in a lossy ground below an air half-space. Triangle denotes the natural frequency for a wire in a homogeneous-space (lossy ground). (b) IE results (Vitebskiy and Carin, 1996) for the natural frequencies of an inclined wire (45 degrees) in a lossy ground below an air half-space.

It can be noted from (35) that for an object in a dielectric half-space near the interface with an air half-space, and in the limit that the dielectric permittivity is large, $R_t \to +1$, and the perturbation is that for a wire in a dielectric near a pmc boundary. This is the dual situation to the case of a wire in a dielectric near a pec boundary, which is approximately the situation considered in Fig. 4. The limiting case of an object in free-space near a pec or pmc boundary was considered in (Baum et al., 1989), which is recovered as a special case of (35).

It was noted previously that the perturbation formula developed here is valid for intermediate values of separation between the object and the nearest material interface. At this point it is beneficial to examine the various approximations utilized in deriving (35). First, in Sec. 3 it was assumed that the material layering has a sufficiently small affect on the object's natural frequency that it may be treated as a perturbation of the homogeneous space result. Subsequently, assuming $s_\alpha \approx s_\alpha^h + \Delta s_\alpha$ where $|\Delta s_\alpha| \ll |s_\alpha^h|$, and expanding the IE kernel in a Taylor's series about s_α^h resulted in the perturbation formula (17). It would be expected that the situation $|\Delta s_\alpha| \ll |s_\alpha^h|$ occurs when $\|\vec{\mathcal{G}}^s\| \ll \|\vec{\mathcal{G}}^h\|$. This would be the case when either $\|\vec{F}\| \ll 1$ in (12), which would arise if the material parameters of the layers were approximately the same as those of the half-space in which the object resides, or the separation parameter 'b' is sufficiently large in (12), or both. Note that the perturbation formula (17) involves Sommerfeld-type integrals (12), although it should be valid for any condition leading to $|\Delta s_\alpha| \ll |s_\alpha^h|$.

Approximate evaluation of (12) leading to the simpler formula (35) was preformed assuming the separation parameter 'b' is sufficiently large such that the integrand is dominated by the rapid variation (decay) of the exponential e^{-pb}. The resulting equation, though, involves the term $e^{-\gamma_\alpha^0 b}$, which becomes unbounded as $b \to \infty$, since $Re\{s_\alpha^0\} \leq 0$. Thus $\lim_{b\to\infty} \Delta s_\alpha \to \infty$, violating the condition $|\Delta s_\alpha| \ll |s_\alpha^h|$. As a result, although the approximate evaluation of the spectral integrals (12) is valid as $b \to \infty$, the perturbation formula is only valid over an intermediate range of values for 'b'. Although spectral integrals were not involved in the formulation detailed in (Baum et al., 1989), similar problems were encountered for $b \to \infty$. The cause of the problem in both situations is that as separation becomes very large, the natural frequencies tend towards the origin in the complex frequency plane and the perturbation procedure based on $|\Delta s_\alpha| \ll |s_\alpha^h|$ is clearly invalid.

Finally, it should be noted that the described spiraling behavior is not found for all objects, or even for all modes of thin wires. It was noted in (Hanson and Baum, 1997) that for two coupled wires, the dominate natural frequency for closely spaced wires, and several other natural modes, tended to spiral about the isolated wire natural frequency. Other natural frequencies of the two wire configuration did not spiral about, but perhaps interacted with, the isolated wire resonance before tending towards the origin for large spacing. It was also shown in (Vitebskiy and Carin, 1996) that for a fat cylinder in the vicinity of an air-ground interface, the spiraling behavior seemed to be absent. While the perturbation formula seems to be applicable for wires and loops/rings in layered media as examined in (Rothwell and Cloud, 1996), (Vitebskiy and Carin, 1996) at this point it is not clear as to what larger class of objects this formula may apply.

6. CONCLUSION

In this paper a perturbation formula is developed which relates the free-space natural frequencies of an object to those of the same object in the presence of a planarly layered medium. The resulting formula involves a numerically computed coefficient which only depends upon the isolated object's characteristics, multiplied by an exponential term which is a function of the separation between the object and the nearest planar interface. The perturbation formula is valid for intermediate spacing between the object and the nearest planar interface. Numerical results are shown for the natural frequencies of a wire in the presence of a layered medium, for several different geometries.

7. ACKNOWLEDGEMENTS

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