Mode-Transformation and Mode-Continuation Regimes on Waveguiding Structures

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***Abstract*—In this paper, modal-interaction phenomena on guided-wave structures are investigated using the theory of critical and singular points. It has been previously shown that classical mode coupling is controlled by the functional characteristics of the dispersion equation in the vicinity of a Morse critical point (MCP), which is real valued for typical structures in the lossless case. The purpose of this study is to demonstrate that two distinct regimes of modal behavior exist in the vicinity of the mode-coupling region, which arise due to the presence of frequency-plane branch points of the dispersion function. These branch-point singularities are intimately associated with the MCP. It is further noted that which of the two regimes governs modal behavior depends on the path of frequency variation or on the presence of loss for time–harmonic problems. Specifically, classical mode coupling is associated with frequency variation between these branch points leading to mode transformation. This traditional mode-transformation behavior is eliminated for the path of frequency variation lying outside of this region resulting in mode continuation (no exchange of physical meaning between modes). The presence of these branch points completely explains the observed phenomena and allows for the conceptualization of the dispersion function in the vicinity of modal interactions.**

***Index Terms*—Branch points, critical points, mode coupling, transmission lines, waveguides.**

I. INTRODUCTION

ELECTROMAGNETIC waveguiding structures are fundamental components of electronic, photonic, and optical systems. A thorough understanding of the characteristics of the discrete propagation modes supported by these structures is important to obtain proper system functionality. Unfortunately, the dispersion characteristics of all but the simplest waveguiding structures must be determined numerically, which obscures the general behavior of the dispersion function. As such, the mathematical cause of modal interaction is not always clear, although its physical cause may often be attributed to a perturbation of some symmetry class.

Since a mathematical model is often needed to predict and quantitatively explain physical phenomena, there has been a considerable amount of research on methods to model modal characteristics, especially mode coupling. The general area of coupled-mode theory [1] has been developed and refined by a large number of authors [2]–[5], and applied to a variety of structures [6]–[13].

A different mathematical model of the mode-coupling phenomena is provided by the theory of Morse critical points (MCP's) [14]–[19], [22]–[24]. It has been found that a Morse point always occurs in the region of mode coupling. In the local vicinity of the Morse point, the dispersion equation is obtained exactly as a quadratic form, which leads to the hyperbolic behavior of the dispersion function in the mode-transformation region. This view of mode transformation was initiated in [14] and [15], and its analytical connection with traditional coupled-mode theory was presented in [19].

While the role of the Morse point in mode-coupling problems has been established, there is another critical point associated with the Morse point, which plays an important role in explaining mode-interaction phenomena. It is demonstrated in this paper that in the vicinity of the nondegenerated Morse point associated with mode coupling, there are always fold-type critical points of the dispersion equation, which define frequency-plane branch points of the dispersion function [21]. These branch points reside off of the real axis in the complex frequency plane for codirectional coupling on lossless structures and, thus, are not encountered in time–harmonic analyses. In the event of material loss, these branch points migrate in the complex frequency plane. As loss is varied, these branch points may cross the real frequency axis, at which point the dispersion behavior is changed in a fundamental manner. It is the purpose of this paper to demonstrate that two distinct propagation regimes exist in the mode-coupling region, one associated with frequency variation between these branch points leading to traditional mode coupling (mode transformation), and one associated with passing outside of this region leading to mode continuation. These ideas allow one to fully conceptualize the behavior of the dispersion function in the region of mode interaction, and to predict the modal behavior as various physical parameters of the structure are changed.

The paper is organized as follows. In Section II, the general theory of MCP's is applied to determine the behavior of the dispersion function in the mode-interaction region. The existence of frequency-domain branch points associated with the Morse point is demonstrated, and their significance is discussed. In Section III, numerical results are shown for two different physical waveguiding structures. The role of the complex frequency-plane branch points in affecting dispersion behavior is accessed, and two distinct mode-interaction regimes are identified.

II. THEORY

Consider a two-dimensional waveguiding structure, invariant along the waveguiding (z) axis. It is assumed the discrete propagation modes that the structure is capable of supporting

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satisfy the *dispersion equation* $H(\kappa, f) = 0$, where κ and f represent the guided-wave normalized propagation constant k_z/k_0 and frequency, respectively, and k_0 is the free-space wavenumber. In this paper, the dispersion equation is generated from a rigorous integral-equation formulation, which is briefly described in Section III. The *dispersion function* for discrete modes is obtained from $H(\kappa, f) = 0$ as $\kappa(f)$. The dispersion function is usually determined implicitly by numerical computation, except for the simplest closed-boundary waveguides where an explicit relationship can be found. The purpose of this paper is to conceptualize the implicit dispersion function $\kappa(f)$ and its associated modal behavior in the vicinity of the mode-coupling region by identifying singularities of $\kappa(f)$ using concepts of critical points (i.e., behavior of derivatives) of the dispersion equation $H = 0$.

We have recently discussed mode-coupling mechanisms on open and shielded perturbed guided-wave structures [19]. It has been shown that hyperbolic-type behavior of characteristic modes is controlled by the presence of a nondegenerate isolated MCP in the mode-transformation region. The functional characteristics of the dispersion equation $H(\kappa, f)$ obtained in the vicinity of the MCP qualitatively and quantitatively reconstruct the modal behavior. These critical (Morse) points are determined as the solution pair (κ_m, f_m) of the system of nonlinear differential equations

$$
\begin{aligned}
H'_\kappa(\kappa, f)|_{(\kappa_m, f_m)} = H'_f(\kappa, f)|_{(\kappa_m, f_m)} = 0 \\
\xi = \left[H''_{\kappa\kappa}H''_{ff} - H''_{\kappa f}H''_{f\kappa}\right]|_{(\kappa_m, f_m)} \neq 0
\end{aligned}
\tag{1}
$$

where the Hessian determinant ξ associated with hyperbolic modal behavior of $H(\kappa, f)$ is strictly negative in the lossless case.

To obtain modal characteristics in the vicinity of the Morse point, we use the Morse lemma [25], [26], which proves that the function H in the vicinity of the MCP can be exactly represented by a quadratic form using a smooth change of coordinates. This leads us to represent the function H local to the Morse point exactly as the Taylor polynomial of order two

$$
\begin{aligned}
H(\kappa, f) = H(\kappa_m, f_m) + \frac{1}{2}H''_{\kappa\kappa}(\kappa - \kappa_m)^2 \\
+ H''_{\kappa f}(\kappa - \kappa_m)(f - f_m) + \frac{1}{2}H''_{ff}(f - f_m)^2
\end{aligned}
\tag{2}
$$

where all partial derivatives are evaluated at (κ_m, f_m).

The local dispersion behavior of propagation constants $\kappa_{1,2} = \kappa_{1,2}(f)$ in the mode-coupling region is represented by the local structure obtained from (2) as

$$
\begin{aligned}
&\kappa_{1,2}(f) \\
&= \kappa_m - \frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f - f_m) \\
&\quad \pm \frac{\sqrt{\left((H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}\right)(f - f_m)^2 - 2H''_{\kappa\kappa}H(\kappa_m, f_m)}}{H''_{\kappa\kappa}}.
\end{aligned}
\tag{3}
$$

Note that the qualitative and quantitative functional behavior ($H(\kappa, f)$ and $\kappa_{1,2}(f)$) in the local vicinity of the MCP (i.e., in the mode-coupling region) is completely determined by (2) and (3).

In [19], we started with (2) and investigated mode coupling by establishing a connection between (3) and the traditional theory of coupled modes. It was shown that (3) leads to a thorough understanding of mode coupling for both the codirectional and contradirectional cases, completely from the standpoint of MCP's via the Morse lemma. In this paper, we again start with (2), but our goal now is to demonstrate that the frequency-domain branch points associated with the square root in (3) and, therefore, intrinsically associated with the Morse point, provide two distinct modal-interaction regimes. The presence of these branch points was not investigated in [19], where the emphasis was on traditional mode coupling. Subsequent to completion of this study, we found similar results concerning the association of branch points singularities with the Morse point in [18] for a shielded resonator problem, for which the dispersion function is available in closed form. It should also be noted that we investigated mode interactions due to the presence of frequency-plane branch points in [20] as well. The branch points investigated in that study are not associated with a Morse point. They are of a completely different origin and possess a different physical significance than those investigated here.

To investigate the influence of the branch points on the mode-interaction behavior, consider the term indicated in (3)

$$
\lambda(f) = \sqrt{\left((H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}\right)(f - f_m)^2 - 2H''_{\kappa\kappa}H(\kappa_m, f_m)}
\tag{4}
$$

which defines a two-valued analytical function in the complex λ-plane. Complex frequency-plane branch points separating branches of the $\lambda(f)$ function and, consequently, the branches of the $\kappa_{1,2}(f)$ function, are determined from (4) as

$$
f_{b1,2} = f_m \pm \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}
\tag{5}
$$

where $H(\kappa_m, f_m) \neq 0$ for the nondegenerate MCP. In the lossless case, the coordinates of the MCP (κ_m, f_m) and the functional characteristics in the local neighborhood of the Morse point $(H(\kappa_m, f_m),\ H''_{\kappa\kappa}(\kappa_m, f_m),\ H''_{ff}(\kappa_m, f_m),\ H''_{\kappa f}(\kappa_m, f_m),\ H''_{f\kappa}(\kappa_m, f_m))$ are obtained as real-valued quantities.

We have shown in [19] that mode coupling of codirectional waves is related to the case $H(\kappa_m, f_m)/H''_{\kappa\kappa} < 0$. The location of branch points $f_{b\,1,2}$ can be immediately obtained from (5) as a complex-conjugate pair of frequency-plane branch points centered about the real-valued Morse frequency point f_m

$$
f_{b1,2} = f_m \pm \jmath\sqrt{\frac{-2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}
\tag{6}
$$

which is illustrated in the insert of Fig. 1. The corresponding values of the propagation constants $\kappa_{1,2}$ at those branch points are easily determined using the form (3)

$$
\kappa_{b1,2} = \kappa_m \mp \jmath\frac{H''_{\kappa f}}{H''_{\kappa\kappa}}\sqrt{\frac{-2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}.
\tag{7}
$$

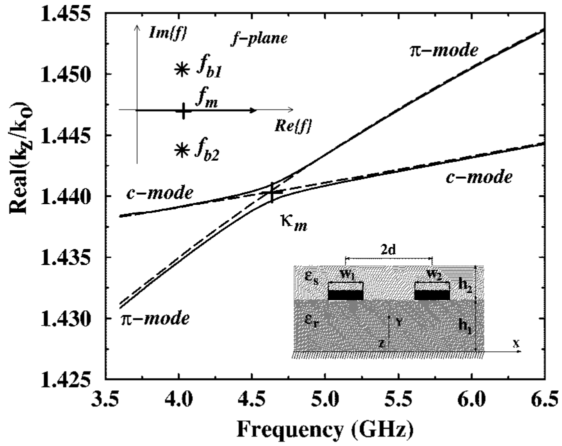


Fig. 1. Dispersion behavior for dominant modes of symmetrical and nonsymmetrical conductor-backed coplanar strip line with infinite superstrate. Degeneracy occurs for the symmetric structure (dashed line: $w_1/h_1 = w_2/h_1 = 0.25$), and is broken by the perturbation of symmetry due to unequal strip widths (solid line: $w_1/h_1 = 0.25$, $w_2/h_1 = 0.27$), $d/h_1 = 0.25$, $h_1 = 1$ cm, $h_2 = 0.1h_1$, $\epsilon_r = 2.25$, $\epsilon_s/\epsilon_r = 1.15$. Insert shows the path of frequency variation.

In addition to the explicit identification of the branch point f_b from (4), it can also be observed [22] that the pair of complex conjugate points $(\kappa_b,\, f_b)$ defined by (6) and (7) represent complex conjugate critical points in the complex $(\kappa,\, f)$ plane satisfying

$$H(\kappa,\, f)|_{(\kappa_b,\, f_b)} = H'_k(\kappa,\, f)|_{(\kappa_b,\, f_b)} = 0 \tag{8}$$

with the nonzero condition

$$\chi = H''_{\kappa\kappa}(\kappa,\, f)H'_f(\kappa,\, f)|_{(\kappa_b,\, f_b)} \neq 0 \tag{9}$$

which is sufficient to guarantee that f_b is a branch point [21].

It can be noted from (6) that a decrease in mode coupling, related to a decrease in the $H(\kappa_m,\, f_m)$ value [19], results in the migration of the complex-conjugate branch points toward the Morse frequency point f_m (equivalently, $(\kappa_b,\, f_b)$ points approach the $(\kappa_m,\, f_m)$ point). The degeneracy of modes (no mode coupling) corresponds to the case when $H(\kappa_m,\, f_m) = 0$ [19] and to the collapse of branch points at the Morse frequency point. We have pointed out in [21] that the Morse point is never a branch point of the dispersion function. Therefore, in the case of mode degeneracy, there are no frequency-plane branch points associated with the MCP. Branch points occur only in the presence of mode coupling, wherein the mode degeneracy is broken with the formation of the characteristic hyperbolic-type behavior.

It can be seen from the preceding discussion and from the numerical results in Section III that interesting features of mode-coupling behavior are associated with branch-point singularities of the dispersion function $\kappa(f)$, which are, in turn, predicted by the existence of critical (Morse) points of the dispersion equation $H = 0$. Characteristic hyperbolic-type behavior in the mode-coupling region is *always* associated with the presence of a nondegenerate MCP, moreover, two frequency-plane branch points *always* accompany the nondegenerate Morse point. Of primary importance here is the migration of the branch points due to the addition of material loss, as will be described in Section III. We will also show how the location of branch points qualitatively affects the mode-coupling behavior.

Although the focus in this paper is on coupling between codirectional waves, a brief description of the role of branch points for contradirectional wave coupling is in order. Mode coupling between forward and backward traveling waves is related to the case when $H(\kappa_m,\, f_m)/H''_{\kappa\kappa} > 0$ [19]. This condition allows for the occurrence of complex modes in the mode-coupling region within the frequency range bounded by the real-valued branch points represented by (5). These branch points $f_{b1,\,2}$ are associated with leaky-wave cutoff frequencies separating complex and complex conjugate solutions of the dispersion function $\kappa_{1,\,2}(f)$ [21]. The expression (5) in conjunction with the normal form representation (3) gives the location of the propagation constants $\kappa_{1,\,2}$ in the κ-plane at the frequency-plane branch points

$$\kappa_{b1,\,2} = \kappa_m \mp \frac{H''_{\kappa f}}{H''_{\kappa\kappa}} \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m,\, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}. \tag{10}$$

The real-valued points $(\kappa_b,\, f_b)$ defined by expressions (5) and (10) in the lossless case are associated with fold points in the $(\kappa,\, f)$ plane governed by (8) and (9). The characteristic behavior in the vicinity of $(\kappa_b,\, f_b)$ is locally determined by the normal form of the fold point [21], [22]

$$\begin{aligned} &(\kappa - \kappa_b)^2 + (f - f_b), &&\text{for } \chi > 0,\\ &(\kappa - \kappa_b)^2 - (f - f_b), &&\text{for } \chi < 0. \end{aligned} \tag{11}$$

In the presence of dielectric-loss MCP's, $(\kappa_m,\, f_m)$ migrate in the complex $(\kappa,\, f)$ plane, and the functional characteristics in the local neighborhood of the MCP become complex-valued quantities. It can be seen that, in this case, the formula (5) corresponds to the complex frequency-plane branch points $f_{b1,\,2}$ located above and below the complex Morse frequency point f_m. Also note that the branch points in the mode-coupling region between codirectional waves in the presence of loss are no longer complex conjugates.

III. Numerical Results and Discussion

To investigate the connection of frequency-plane branch points with mode-coupling behavior, examples of a conductor-backed coplanar strip line with infinite superstrate and a grounded dielectric slab with anisotropic chirality will be considered [19]. In each case, a rigorous full-wave solution has been used to generate the dispersion equation $H(\kappa,\, f) = 0$. In the first example (geometry shown in the inset of Fig. 1), an electric-field integral equation similar to [22] and [27] is formed by enforcing the boundary condition for the tangential components of the electric field on the surface of the conducting strips. The correctness of the full-wave code has been checked extensively by comparison to results in the literature (e.g., [27], [28], among others). In the second example, dispersion behavior of guided surface-wave modes on lossy grounded dielectric slabs with anisotropic chirality has been investigated via the volume equivalence principle for bianisotropic media [29]. The numerical code was checked by comparison to

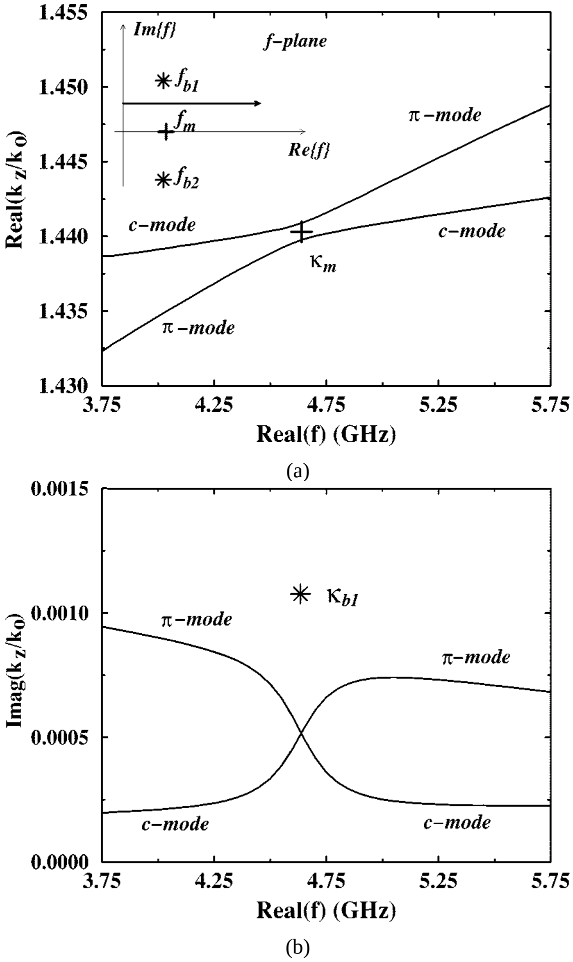


Fig. 2. Propagation constant characteristics for dominant modes in nonsymmetrical conductor-backed coplanar strip line versus complex frequency with $\mathrm{Im}\{f\} = 0.1$ GHz. Insert shows the path of frequency variation.

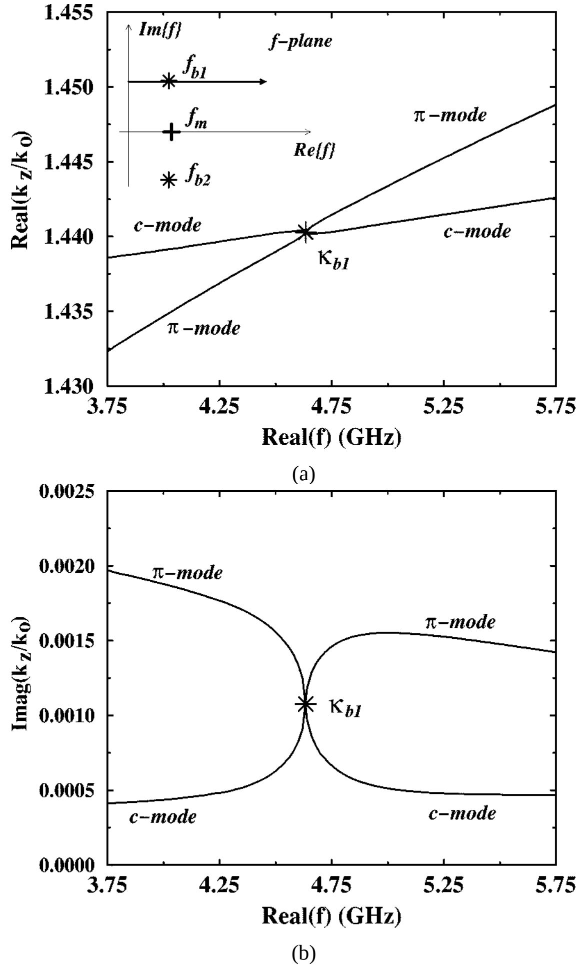
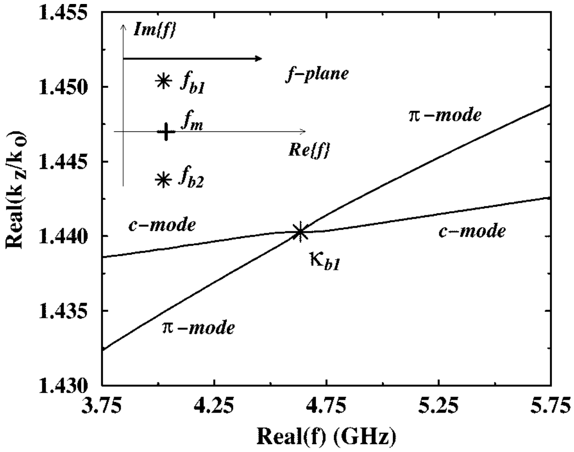


Fig. 3. Propagation constant characteristics for dominant modes in nonsymmetrical conductor-backed coplanar strip line versus complex frequency with $\mathrm{Im}\{f\} = 0.2083$ GHz. A bifurcation of characteristic curves occurs at the κ_{b1} point when the path of frequency variation crosses the branch point f_{b1} in the complex f-plane.

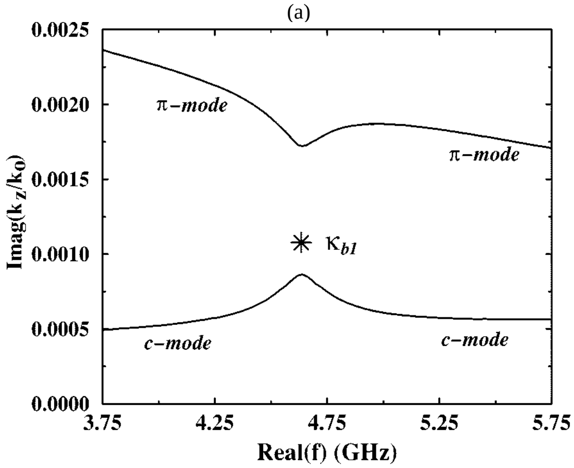
isotropic chiral slab results [30] and anisotropic dielectric slab results [31]. Note that chirality is used here simply to induce mode coupling in the slab waveguide. In both cases, the full-wave results were used to generate $H(\kappa, f)$; the various derivatives on H, introduced in (1), (8), and (9), were calculated via finite-difference methods. The characteristic determinant $H(\kappa, f)$ is obtained implicitly (no closed-form solution exists for these particular problems) and it is calculated numerically. Note that coordinates of MCP's (κ_m, f_m) and branch-point pairs (κ_b, f_b) are determined as the numerical solution of (1), (8), and (9), respectively, using a numerical root search. In this procedure, H is given numerically at each step of the iteration algorithm of the root search and in the calculation of finite-difference approximations.

In the first part of the discussion, a conductor-backed coplanar-strip line is considered in the absence of dielectric loss. It is shown that the path of frequency variation with respect to the location of the complex branch points affects the mode-coupling behavior. In the second part, we will discuss the migration of branch points in the complex frequency plane versus dielectric loss for the above-mentioned structures, and the influence of these branch points on the mode-coupling behavior for time–harmonic (real-valued) frequencies.

It has been discussed in [19] for the example of conductor-backed coplanar strips that changes in strip width of one conductor $(w_1 \neq w_2)$ perturb the symmetry of the structure (inset in Fig. 1), leading to the transformation of uncoupled odd and even modes (which possess a degeneracy) into coupled nondegenerate c and π modes, as shown in Fig. 1. The nondegenerate MCP with coordinates $(\kappa_m, f_m) = (1.4403, 4.638)$ is obtained in the mode-coupling region of the perturbed structure. A numerical root search has been performed for the branch-point pair (κ_b, f_b) associated with the MCP, satisfying (8) and (9). Numerically determined values were found to be $(\kappa_{b1,2}, f_{b1,2}) = (1.4403 \pm \jmath 0.001\,08, 4.6338 \pm \jmath 0.2083)$. Coordinates of the propagation constant and frequency-plane branch points $f_{b1,2}$ have been also calculated using (6) and (7), resulting in $(\kappa_{b1,2}, f_{b1,2}) = (1.4403 \pm \jmath 0.001\,075, 4.638 \pm \jmath 0.2081)$, showing good agreement with the numerical results of the root search. The κ_b values are not shown in Fig. 1 since $\mathrm{Re}\{\kappa_m\}$ of the Morse point and $\mathrm{Re}\{\kappa_b\}$ of the branch points are practically identical for small coupling. The inset in Fig. 1 shows the path of frequency variation along the real axis in the complex frequency plane, with the f_b values denoted by f_{b1} and f_{b2}.



(a)



(b)

Fig. 4. Propagation constant characteristics for dominant modes in nonsymmetrical conductor-backed coplanar strips versus complex frequency with $\mathrm{Im}\{f\} = 0.25$ GHz. No mode transformation occurs when the frequency variation path is deformed above the branch point f_{b1} in the complex f-plane.

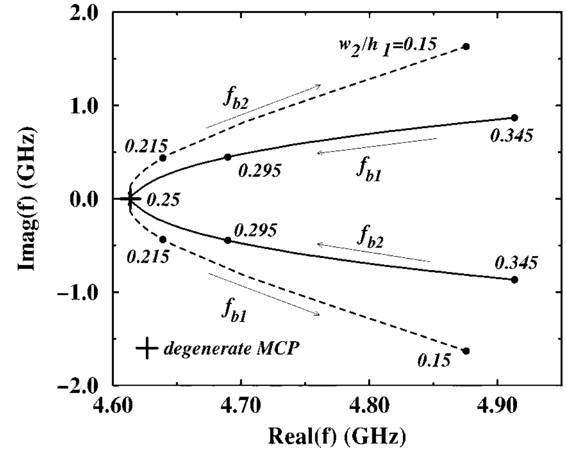


Fig. 5. Evolution of complex conjugate frequency-plane branch points parameterized by strip width w_2/h_1.

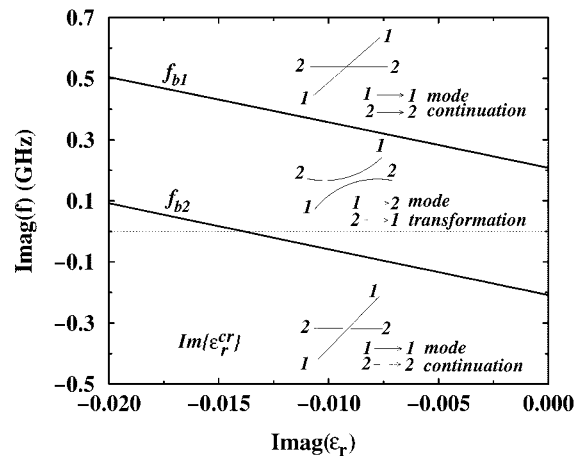


Fig. 6. Evolution of complex frequency-plane branch points parameterized by dielectric loss in nonsymmetrical conductor-backed coplanar strip line (see inset in Fig. 1). $d/h_1 = 0.25$, $w_1/h_1 = 0.25$, $w_2/h_1 = 0.27$, $h_1 = 1$ cm, $h_2 = 0.1h_1$, and $\epsilon_s/\mathrm{Re}\{\epsilon_r\} = 1.15$.

To investigate the role of the branch points f_b, the path of frequency variation has been deformed into the complex frequency plane. Fig. 2 demonstrates the dispersion behavior of dominant modes in the mode-coupling region versus complex frequency with $\mathrm{Im}\{f\} = 0.1$ GHz. It can be seen that the qualitative hyperbolic-type behavior is not affected by the deformation of the frequency variation path into the first quadrant of the complex f-plane. It turns out that this is true for any path passing between the conjugate branch points. For any frequency path between these branch points, transformation of the c to π mode and the π to c mode occurs as it does for real-frequency variation. We call this the *mode transformation regime* since physical attributes of the mode (e.g., the c-mode is quasi-odd while the π-mode is quasi-even) are interchanged between dispersion curves in passing through this region.

The dispersion behavior of dominant modes is qualitatively changed if the path of frequency variation crosses the branch point (Fig. 3). This type of bifurcation has been discussed in [21] in connection with the analysis of leaky-wave cutoff frequencies in open-boundary waveguides. Note that no conclusions can be made in this case regarding transformation or continuation among modes. It is important to emphasize that, in passing through the branch point, we break the hyperbolic-type universal unfolding of the MCP associated with the mode-coupling behavior [19], [24]. Similar qualitative behavior of dominant modes occurs if the path of frequency variation crosses the fourth quadrant branch point f_{b2} in the complex f-plane.

If the path of frequency variation is deformed above f_{b1} in the complex f-plane, as shown in Fig. 4, or below the fourth quadrant branch point f_{b2}, the mode transformation characteristic hyperbolic behavior no longer occurs, resulting in the continuation of the modes π to π and c to c. We call this the *mode continuation regime* since physical meaning of each dispersion curve (c or π) remains unchanged in passing through this region. Operation in this regime will significantly effect the operation of devices that rely on the exchange of modes for correct operation.

As we continue to deform the frequency path in the first or fourth quadrant of the complex f-plane even further above or below the branch points, the characteristic curves of the dominant modes approach those obtained for the symmetrical structure (which has a degenerate rather than a nondegenerate Morse point, and which does not exhibit frequency-plane branch points).

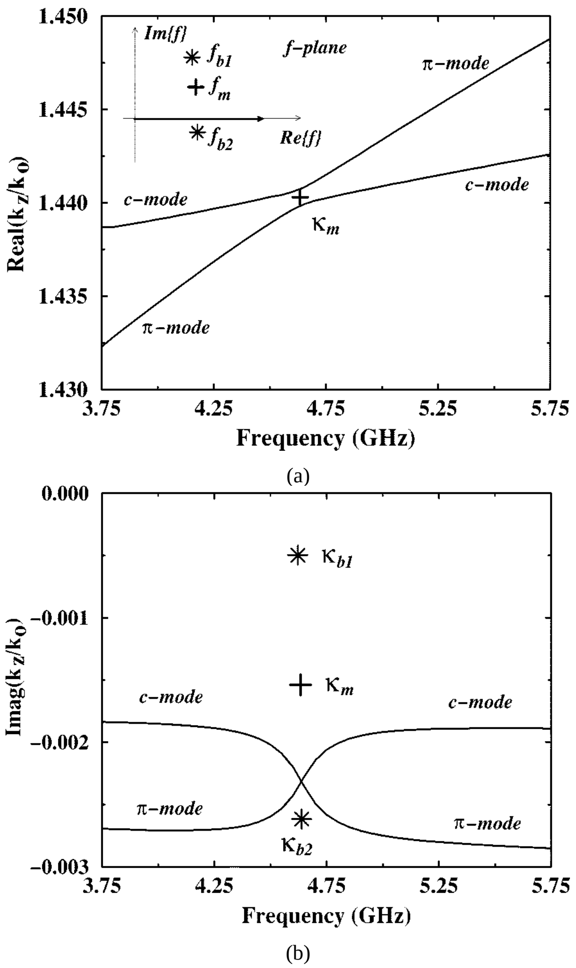


Fig. 7. Dispersion behavior of dominant modes in lossy nonsymmetrical conductor-backed coplanar strip line with $\mathrm{Im}\{\epsilon_r\} = -0.01$. The path of frequency variation lies between branch points f_{b1} and f_{b2}. Critical points $(\kappa_{b1}, \kappa_{b2}, \kappa_m)$ are shown in the complex (κ, f) plane.

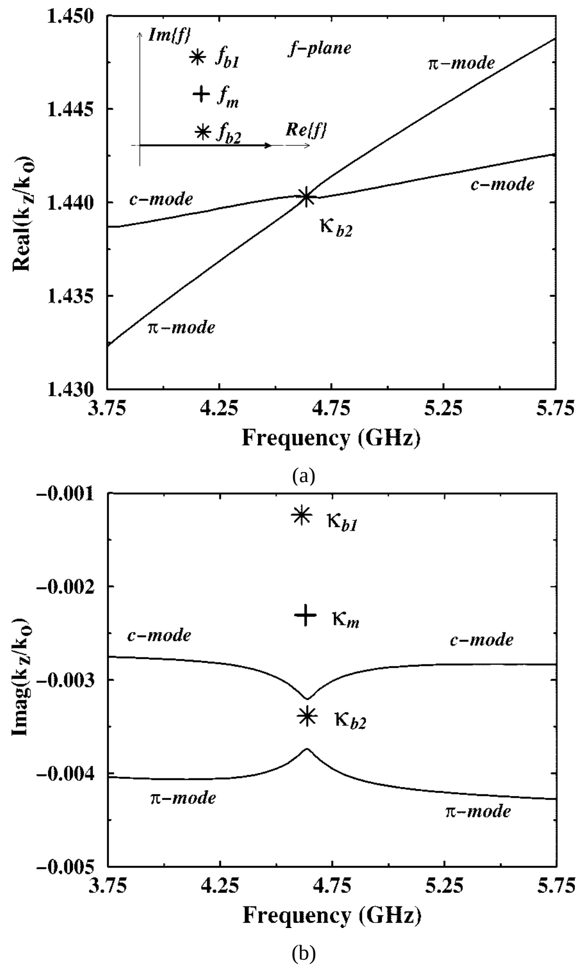


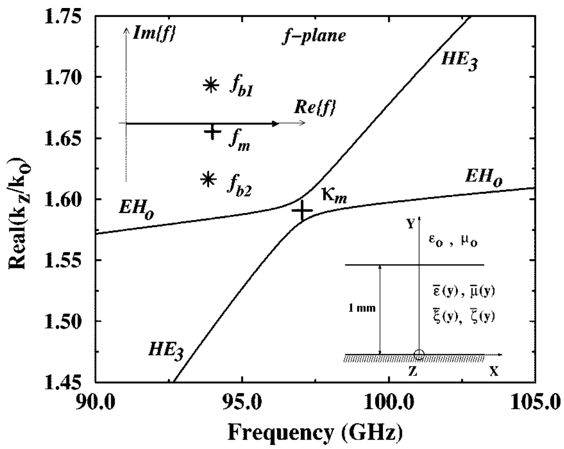
Fig. 8. Dispersion behavior of dominant modes in lossy nonsymmetrical conductor-backed coplanar strip line with $\mathrm{Im}\{\epsilon_r\} = -0.015$. The path of frequency variation lies below the branch points due to migration of f_{b2} into the first quadrant of the complex f-plane.

The above illustrative analysis leads to the conclusion that the characteristic hyperbolic-type mode-coupling behavior associated with mode transformation occurs only if the path of frequency variation lies within the strip between complex-conjugate frequency-plane branch points. Mode continuation occurs if the path is deformed above f_{b1} or below f_{b2}.

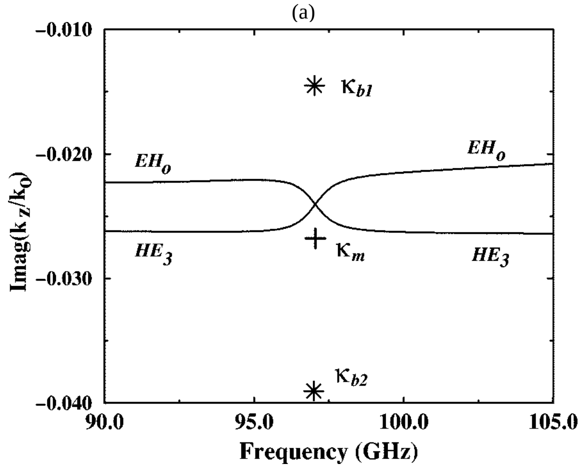
The evolution of complex-conjugate frequency-plane branch points parameterized by strip width is demonstrated in Fig. 5 for the example of a conductor-backed coplanar strip line. The first quadrant branch point f_{b1} and the fourth quadrant branch point f_{b2} approach the degenerate Morse frequency point f_m as strip width w_2 approaches w_1 $(w_2 > w_1)$. In the symmetrical case $(w_1 = w_2, H(\kappa_m, f_m) = 0)$, those branch points collapse due to the absence of mode coupling. They continue to migrate in opposite quadrants as strip width w_2 is decreased with respect to w_1.

While frequency variations in the complex plane are nonphysical for time–harmonic problems, the above analysis is intended to demonstrate that two distinct regimes exist (mode transformation or mode continuation) depending on the path of frequency variation relative to the branch-point locations. We will investigate the migration of complex frequency-plane branch points versus dielectric loss, and their influence on the mode coupling behavior for time–harmonic (real-valued) frequencies in the following. The example of a nonsymmetrical conductor-backed coplanar strip line with infinite superstrate is considered in the presence of a lossy substrate with $\epsilon_r = 2.25 - \jmath\,\mathrm{Im}\{\epsilon_r\}$.

It is noted here that complex frequency-plane branch points pairs (complex conjugate in lossless structures) migrate versus dielectric loss, such that at some critical value of loss one of the branch points crosses the real frequency axis (f_{b2} in this particular example, as shown in Fig. 6). It can be seen from Fig. 6 that for any value of loss greater than the critical value $\mathrm{Im}\{\epsilon_r^{cr}\}$, the path of frequency variation along the real frequency axis lies below both branch points f_{b1} and f_{b2}. Passing below (above) or between the branch point pair while varying frequency along the real axis has the same effect as that noted for the lossless case for complex-valued frequency paths. Therefore, a critical value of loss separates two qualitatively different states of the structure associated with mode transformation and mode continuation. Any path of frequency variation (along the real axis for time–harmonic problems) that lies between f_{b1} and f_{b2} guarantees the formation of hyperbolic-type mode-transformation behavior. Frequency variation along the real axis that passes above f_{b1} (i.e., f_{b1} and f_{b2} lie in the fourth quadrant of the frequency

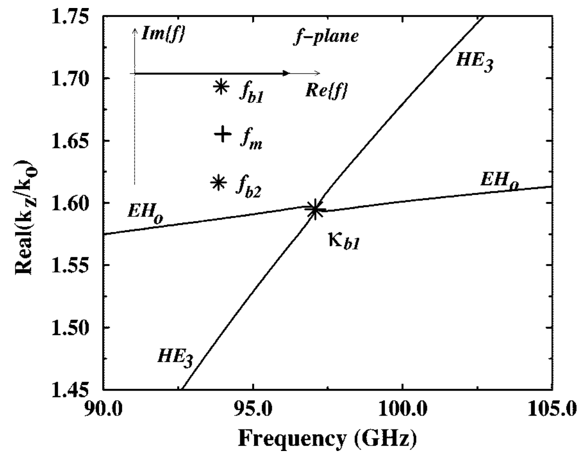


(a)

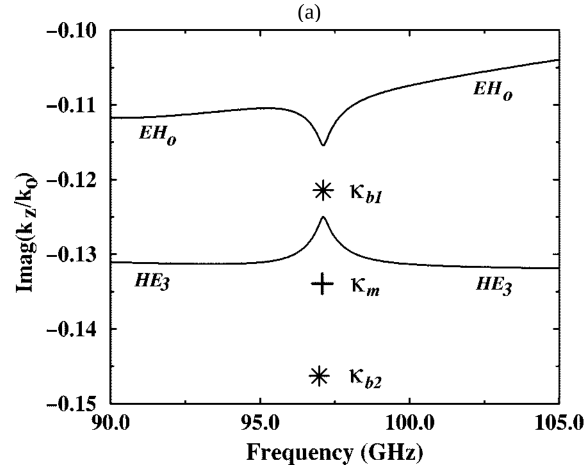


(b)

Fig. 9. Dispersion behavior of guided surface-wave modes on lossy grounded dielectric slab waveguide with anisotropic chirality: $\epsilon_{xx} = 9\epsilon_0$, $\epsilon_{yy} = \epsilon_{zz} = 3\epsilon_0$, $\mu_{xx} = \mu_{yy} = \mu_{zz} = \mu_0$, $\xi_{\alpha\beta} = -\zeta_{\alpha\beta} = j\kappa_{\alpha\beta}\sqrt{\epsilon_0\mu_0}$, $\kappa_{xx} = 0.05$, $\kappa_{yy} = \kappa_{zz} = 0$, $\mathrm{Im}\{\epsilon_r\} = -0.09$ (for all permittivity components). The path of frequency variation is between branch points f_{b1} and f_{b2} in the complex f-plane.



(a)



(b)

Fig. 10. Dispersion behavior of guided surface-wave modes on lossy grounded dielectric slab waveguide with anisotropic chirality for $\mathrm{Im}\{\epsilon_r\} = -0.45$ (for all permittivity components). Branch points migrate into the fourth quadrant of the f-plane, resulting in a qualitative change in the modal behavior.

plane) or below f_{b2} (i.e., f_{b1} and f_{b2} lie in the first quadrant) results in mode continuation, as indicated in Fig. 6. Under which regime the structure operates depends on the value of loss.

To illustrate the occurrence of mode transformation for the lossy case, a value of dielectric loss $\mathrm{Im}\{\epsilon_r\} = -0.01$ is examined, which provides the location of branch points f_{b1} and f_{b2} in opposite quadrants with respect to the path of time–harmonic frequencies (Fig. 7). The coordinates of the Morse frequency point and branch points have been calculated numerically as the solution of nonlinear equations given by (1), (8), and (9), respectively, resulting in $f_m = 4.634 + j0.1503$, $f_{b1} = 4.619 + j0.356$, and $f_{b2} = 4.638 - j0.058$. The characteristic mode coupling behavior and the transformation of the c and π modes is shown in Fig. 7 for time–harmonic frequencies.

To demonstrate that modal interchange ceases when stronger losses are introduced, consider an increase of the dielectric loss to $\mathrm{Im}\{\epsilon_r\} = -0.015$, which results in the migration of all critical and branch points, especially the branch point f_{b2}, across the real frequency axis into the first quadrant of the complex f-plane. The coordinates of the Morse frequency point and branch points at this value of loss become $f_m = 4.629 + j0.225$, $f_{b1} = 4.612 + j0.432$, $f_{b2} = 4.637 + j0.017$. The qualitative behavior of the dispersion function is dramatically changed, as shown in Fig. 8. The mode transformation shown in Fig. 7 is changed into mode continuation (Fig. 8). As such, for physical time–harmonic frequencies, a smooth change of dielectric loss may result in significant qualitative changes in the mode coupling behavior, which is associated with migration of complex frequency-plane branch points across the real frequency axis.

To provide another example, dispersion behavior of guided surface-wave modes on a lossy grounded dielectric slab waveguide with anisotropic chirality [see inset in Fig. 9(a)] is investigated in connection with complex frequency-plane branch points. Fig. 9 demonstrates the hyperbolic type behavior of hybrid EH_0 and HE_3 modes in the lossy anisotropic chiral slab with $\mathrm{Im}\{\epsilon_r\} = -0.09$ (dielectric loss for all permittivity components) and their mutual transformations. The path of time–harmonic frequencies lies between complex branch points f_{b1} and f_{b2} with coordinates $f_{b1} = 97.0227 + j0.5863$ GHz and $f_{b2} = 96.995 - j0.9199$ GHz, respectively. The MCP has been found in the mode-coupling region with $f_m = 97.0512 - j0.1678$ GHz. These special points of interest migrate into the fourth quadrant of the complex f-plane as loss is increased, as shown in Fig. 10. At some critical value of $\mathrm{Im}\{\epsilon_r\}$, the branch point f_{b1} crosses the real frequency axis resulting in mode bifurcation. For the case demonstrated in Fig. 10 ($\mathrm{Im}\{\epsilon_r\} = -0.45$), the path of frequency variation is above the branch points and the Morse point with coordinates $f_{b1} = 97.1152 - \jmath 0.0755$, $f_{b2} = 96.9781 - \jmath 1.5922$, $f_m = 97.0883 - \jmath 0.8387$. A migration of the f_{b1} in the fourth quadrant across the real frequency axis leads to the formation of the modal continuation behavior with no mode transformation.

IV. Conclusion

Modal interactions on waveguiding structures have been investigated using the theory of critical and singular points. It has been shown that frequency-plane branch points of the dispersion function exist in the mode-coupling region. These branch points are associated with the occurrence of a nondegenerate MCP, which has been previously shown to occur in the mode-coupling region and which is connected with traditional coupled-mode theory. The identified branch points do not influence modal behavior for codirectional coupling in time–harmonic lossless problems, but may significantly affect mode-interaction phenomena in the event of material loss. Numerical examples have been shown for two different waveguiding structures, although the effect is general in nature.

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