Leaky-Wave Dispersion Behavior on a Grounded Ferrite Slab Waveguide

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***Abstract*—In this letter, leaky-wave dispersion behavior is studied for a grounded biased-ferrite slab waveguide. The full-wave analysis encompasses space-wave leaky modes, surface waves, and magnetostatic modes supported by a ferrite slab as the magnetic bias field varies in strength. In particular, as the bias field increases from a null value the TE leaky-wave cutoff frequencies of the isotropic slab split into two cutoff frequencies, resulting in complicated dispersion behavior of the corresponding forward and backward leaky waves. TM modes are unaffected by variation in the strength of the bias field for this orientation.**

***Index Terms*—Ferrite slab, leaky waves, surface waves.**

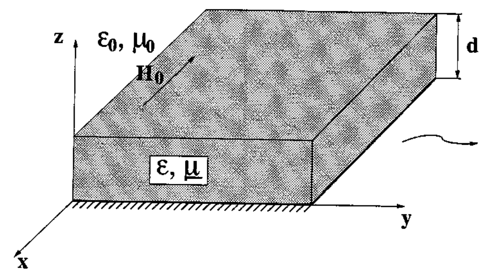

Fig. 1. Grounded ferrite slab waveguide with applied magnetic bias field. Wave propagation is in the positive y-direction.

I. INTRODUCTION

BIASED ferrites play a significant role in various nonreciprocal devices such as phase shifters, polarizers, and isolators, where the electromagnetic properties of such devices can be controlled by varying the applied magnetic bias field [1]–[3]. Magnetostatic and surface waves supported by ferrite structures have been of primary interest for a long time. Magnetostatic and surface waves of a biased ferrite slab were analyzed in [4]–[7], and magnetostatic volume and surface waves on strip and slot printed lines were recently investigated in [3], [8]. There has also been some work on the analysis of leaky waves on ferrite slabs in the form of radiating space-wave modes [9], [10].

In this paper, we present a study of space-wave leaky modes on a grounded ferrite slab waveguide, biased with a transversely applied magnetic field (Fig. 1). The analysis is based on the numerical solution of an integral equation, formulated for an equivalent volume current density [11], and it uses principles of singularity theory. In recent papers, we have presented a method based on singularities and critical points which helps to explain observable modal phenomena. In [12], singular and critical points were identified in connection with different leaky-wave regimes on a coplanar strip line, and in [13], singular points and associated frequency-plane branch points were shown to govern modal behavior in the vicinity of cutoff in a variety of transmission line and waveguiding structures.

For an isotropic dielectric slab, proper surface and improper leaky modes are relatively well understood. This is not the case for the corresponding modes on a biased ferrite slab, on which the modes are considerably more complicated. In this letter, we study the evolution of proper surface and improper leaky modes as a grounded dielectric slab is smoothly changed into a biased ferrite, to illustrate how the mode spectrum of the biased ferrite waveguide evolves, and how it is related to the simpler isotropic slab waveguide.

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II. THEORY

Considering the two-dimensional ferrite planar waveguiding structure depicted in Fig. 1, which is invariant along the ρ-axis ($\rho = \sqrt{x^2+y^2}$), and subsequent to a three-dimensional Fourier transform in space and time, $(x, y, t) \longleftrightarrow (\xi, \lambda, \omega)$, the integral equation formulation developed in [11] for vertically inhomogeneous bianisotropic media is reduced to an operator equation for the discrete modes of the structure

$$A\left(\lambda, \omega, \varepsilon, \underline{\mu}, d\right) X = 0. \tag{1}$$

In (1), A is an operator-function, λ is the spatial Fourier-transform variable representing the modal propagation constant ($\xi = 0$, indicating no mode variation in the x-coordinate), ω is the temporal Fourier-transform variable representing angular frequency, and X represents a modal field distribution. It is assumed that each of the variables (λ, ω) can be analytically continued into the complex plane, and that $\varepsilon\,(\varepsilon = \varepsilon_0\varepsilon_r)$, $\underline{\mu}$, and d have specified values. The dyadic permeability $\underline{\mu}$ is given by

$$\underline{\mu} = \mu_0 R(\theta, \phi) \begin{bmatrix} \mu & \jmath\kappa & 0 \\ -\jmath\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} R^T(\theta, \phi) \tag{2}$$

where

$$R(\theta, \phi) = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix} \tag{3}$$

represents a rotation matrix which fixes the position of the optical axis and R^T is the transpose of R. In (2), $\mu = 1 + (\omega_0\omega_M)/(\omega_0^2 - \omega^2)$, $\kappa = \omega\omega_M/(\omega_0^2 - \omega^2)$,

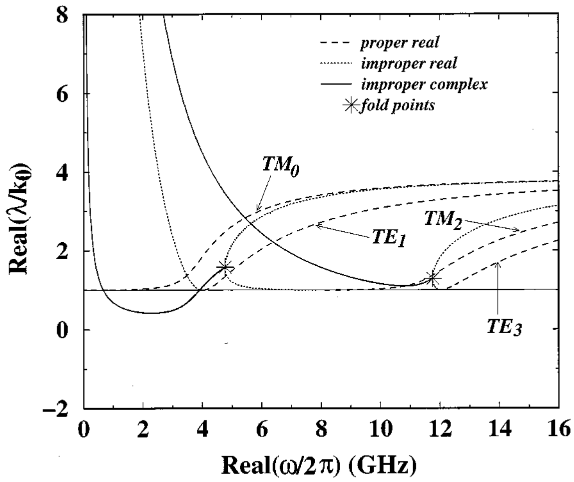

Fig. 2. Dispersion behavior of the lower-order TE and TM surface waves in a grounded dielectric slab waveguide with d = 0.5 cm and ϵ_r = 15.

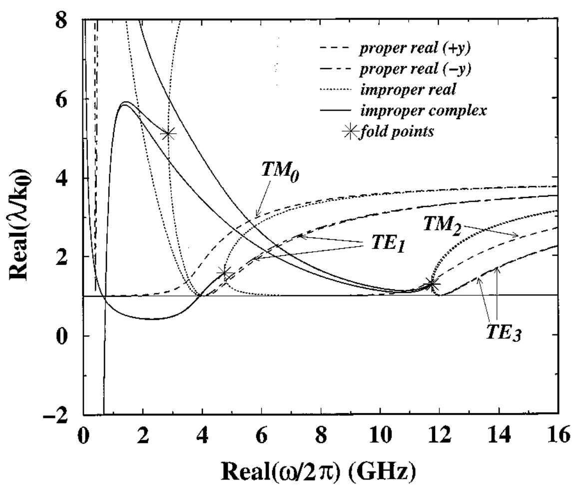

Fig. 3. Dispersion behavior of magnetostatic, TE and TM surface waves, and associated space-wave leaky modes on a grounded ferrite slab waveguide with H_0 = 10 G and $\mu_0 M_s$ = 100 G.

$\omega_0 = \gamma\mu_0 H_0$, $\omega_M = \gamma\mu_0 M_s$, H_0 is the dc magnetic bias field, M_s is the material saturation magnetization, and $\gamma = -1.759 \times 10^{11}$ kg/coul. In the following, we will set $\mu_0 M_s = 10 H_0$ and vary H_0.

Nontrivial solutions of (1) are obtained from the implicit dispersion equation

$$H\left(\lambda,\, \omega,\, \varepsilon,\, \underline{\mu},\, d\right) = \det\left(A\left(\lambda,\, \omega,\, \varepsilon,\, \underline{\mu},\, d\right)\right) = 0 \qquad (4)$$

where one fixes ω, ε, $\underline{\mu}$, and d and determines λ by a numerical root-search. More generally, H maps $(\lambda,\, \omega,\, \varepsilon,\, \underline{\mu},\, d)$ into the complex plane **C**. A study of the properties of the mapping H leads to the analysis of singularities associated with the complex variables $(\lambda,\, \omega)$ which explain various modal phenomena. For the structure considered here, we are particularly interested in determining the set of singular (fold) points of the mapping H, which give the leaky-wave cutoff frequencies of the modes [13], defined by

$$H(\lambda,\, \omega) = \frac{\partial H(\lambda,\, \omega)}{\partial \lambda} = 0, \qquad \frac{\partial^2 H(\lambda,\, \omega)}{\partial \lambda^2}\, \frac{\partial H(\lambda,\, \omega)}{\partial \omega} \neq 0. \qquad (5)$$

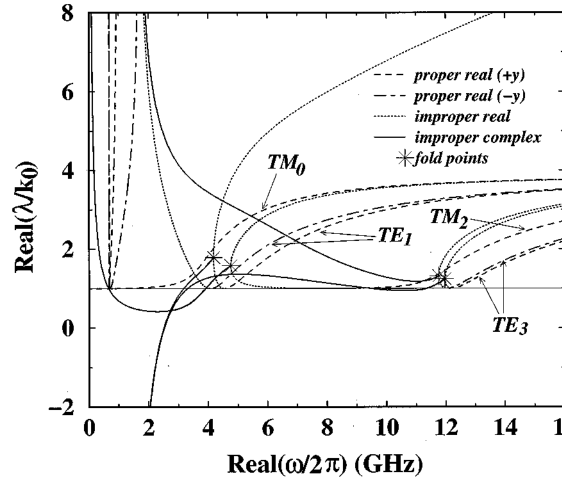

Fig. 4. Dispersion behavior of magnetostatic, TE and TM surface waves, and associated space-wave leaky modes on a grounded ferrite slab waveguide with H_0 = 50 G and $\mu_0 M_s$ = 500 G.

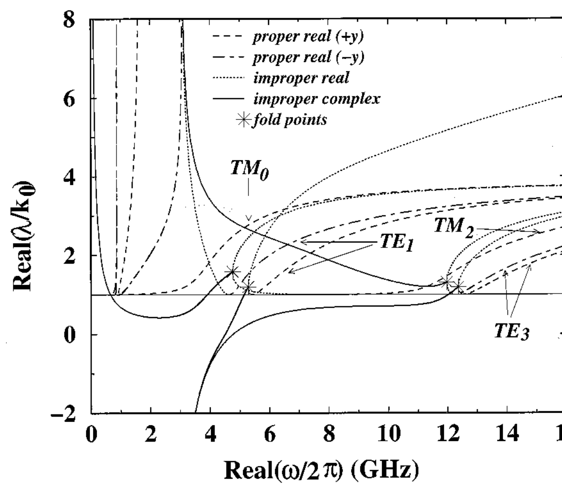

Fig. 5. Dispersion behavior of magnetostatic, TE and TM surface waves, and associated space-wave leaky modes on a grounded ferrite slab waveguide with H_0 = 100 G and $\mu_0 M_s$ = 1000 G.

III. Numerical Results and Discussions

Numerical results for magnetostatic, TE and TM surface waves, and associated space-wave leaky modes supported by a grounded slab waveguide (Fig. 1) are shown in Figs. 2–5. In Fig. 2, surface and leaky modes on an isotropic dielectric slab waveguide ($H_0 = M_s = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0\varepsilon_r$; in all figures $d = 0.5$ cm and $\epsilon_r = 15$) are shown. The identification of fold point singularities provide the leaky-wave cutoff frequencies. Note that only the TM_2 mode is a fast $(\lambda/k_0 < 1)$ leaky mode. In Fig. 3, a small magnetic bias field is introduced (H_0 = 10 G, $\mu_0 M_s = 10 H_0$), resulting in an anisotropic permeability and nonreciprocal mode propagation (i.e., a ferrite material). It is interesting to note that the TE_1 mode is never leaky on the isotropic slab (Fig. 2), yet bifurcates into a nonleaky backward wave and a leaky forward wave (with an associated fold point), as the material becomes a ferrite (Fig. 3). In an isotropic slab, this fold point resides at infinity, and in the presence of the magnetic bias field migrates as shown in Fig. 6. The leaky-wave cutoff frequency (fold point) for each TE_n

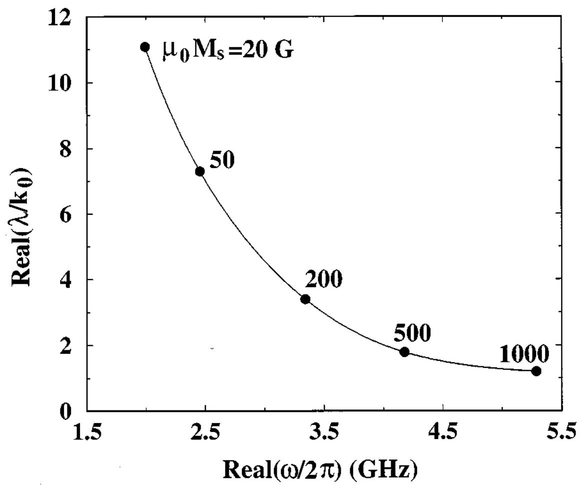

Fig. 6. Migration of the leaky-wave cutoff of the TE_1 forward wave parameterized by the applied bias magnetic field.

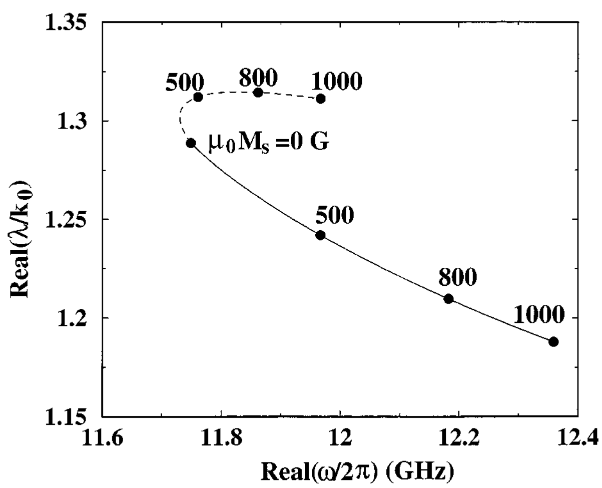

Fig. 7. Migration of the leaky-wave cutoff of the TE_3 forward and backward waves parameterized by the applied bias magnetic field.

mode, $n > 1$, bifurcates into two cutoff frequencies (fold points), one for the backward mode and one for the forward mode (for the TE_3 mode in Fig. 3 they are very close together). The magnetostatic modes are the nearly vertical lines on the left-hand side of the figure.

In Figs. 4 and 5, the bias field is increased ($\mu_0 M_s = 10H_0$ is maintained), enhancing the nonreciprocal nature of the mode propagation and affecting the dispersion of the magnetostatic modes. For sufficiently large bias field, the forward TE_3 leaky mode is fast $(\lambda/k_0 < 1)$ over a large frequency span, and, hence, may significantly contribute to radiation. Note that the transverse bias field (as shown in Fig. 1) affects only the TE modes, although the TM modes are also shown in Figs. 2–5 for completeness. For the usual case of a strongly biased ferrite (Fig. 5, $H_0 = 100$ G) the dispersion curves of magnetostatic and surface waves are in excellent agreement with those published in the literature (e.g., see [6, Fig. 2]).

The evolution of the leaky-wave cutoff of the TE_3 mode parameterized with a magnetic bias field is shown in Fig. 7 (solid line: forward mode, dashed line: backward mode). It can be seen that an increase in the applied magnetic field separates further the leaky-wave cutoff points associated with forward and backward waves, and significantly affects the dispersion behavior of these modes away from the cutoff frequency.

IV. Conclusion

In this paper, we have presented a study of the evolution of space-wave leaky modes as a grounded isotropic slab waveguide is smoothly transformed into a ferrite slab waveguide. The analysis of the modal spectrum is based on a full-wave formulation in conjunction with singularity theory. It was observed that as the slab transitions into a ferrite each leaky-wave cutoff frequency of the TE_n waves, $n > 1$, associated with a fold singular point, splits into two isolated fold points corresponding to forward and backward waves, resulting in the complicated nonreciprocal dispersion behavior of space-wave leaky modes. The analysis of singularities of the dispersion function for ferrite structures reveals interesting modal effects associated with the nonreciprocal nature of the material.

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