Integral Equation Formulation for Inhomogeneous Anisotropic Media Green's Dyad with Application to Microstrip Transmission Line Propagation and Leakage

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***Abstract*—A straightforward numerical technique based on the equivalence principle is presented to determine the complete spectral Green's dyad for inhomogeneous anisotropic media. This method is relevant to guided-wave problems where propagation characteristics are desired in the axial transform domain. Spectral Green's components are determined from a one-dimensional polarization-type integral equation. This method is very simple and versatile, and can be used to model continuously varying or stratified dielectric media with permittivity dyads of the most general form. As an application, a microstrip transmission line residing on a generally orientated uniaxial and biaxial substrate is considered, and new results for higher-order mode leakage are presented.**

I. Introduction

The study of waveguiding structures embedded in anisotropic media is important for several reasons. Materials used in the construction of microwave and millimeter-wave circuits may exhibit naturally occurring or material processing related dielectric anisotropy [1]. Also, in some applications introducing anisotropy can enhance circuit performance or provide some added flexibility and control over circuit behavior [2]. For the case of axially-invariant waveguiding structures, it is important to be able to predict the effect that intentional or unintentional anisotropy has on propagation and leakage characteristics.

Integral equation (IE) techniques are very popular for determining propagation characteristics of transmission lines. The IE method provides physical insight, yields accurate results, and is usually straightforward once the Green's function for the surrounding environment is determined. Analytical determination of the Green's function for multilayered anisotropic media is extremely complicated, especially for biaxial or misaligned uniaxial media, and often seminumerical techniques are employed. A few references that are particularly relevant to this work are made here. In [2], Green's function components for a single grounded biaxial or uniaxial slab are determined analytically. The two-layered case for a misaligned uniaxial medium is described in [3]. In [4], [5], a matrix approach is applied to study general anisotropic, multilayered media, and in [6], [7], a matrix approach is formulated for the most general bianisotropic, multilayered media.

The above methods are quite complicated analytically, with the more general matrix methods requiring some numerical effort as well. A simple numerical method for determining the complete Green's dyad is proposed here which accommodates multiple layers easily, allows for a full complex-valued matrix representation of the permittivity dyad, and yields results that are analytically integrable in directions transverse to the waveguiding axis. The method follows from a polarization-type IE using the relatively simple Green's function for two isotropic homogeneous half-spaces. As an application, this technique is applied to the study of microstrip transmission lines on anisotropic substrates, operating in both the bound and leaky regimes. Results for microstrip leakage involving anisotropic substrates are also presented in [8].

II. Theory

As motivation for determining the Green's dyad for inhomogeneous anisotropic media, an open microstrip transmission line geometry is shown in Fig. 1. The region $z < 0$ can be an isotropic dielectric or a conductor. The integral equation for the unknown natural-mode surface current on the line with an assumed dependence of $e^{-j(k_y y - \omega t)}$ is

$$\hat{t} \cdot \int_l \overleftrightarrow{g}_e^a(\vec{\rho} \mid \vec{\rho}\,'; k_y) \cdot \vec{K}(\vec{\rho}\,')\, dl' = 0 \qquad \cdots \forall \vec{\rho} \in l \tag{1}$$

where $k_y = \beta - j\alpha$ is the unknown complex propagation constant, $\vec{K}(\vec{\rho})$ is the unknown natural-mode surface current, and $\hat{t}$ is a unit tangent to the microstrip line. The electric Green's dyad, $\overleftrightarrow{g}_e^a$, rigorously accounts for the inhomogeneous anisotropic media surrounding the transmission line, and can be written as

$$\overleftrightarrow{g}_e^a(\vec{\rho} \mid \vec{\rho}\,'; k_y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overleftrightarrow{g}_e^a(k_x, k_y, z \mid z') e^{jk_x(x-x')} dk_x \tag{2}$$

where

$$\overleftrightarrow{g}_e^a(k_x, k_y, z \mid z') = \mathrm{PV}\overleftrightarrow{\mathrm{g}}^a(\mathrm{k_x}, \mathrm{k_y}, \mathrm{z} \mid \mathrm{z}') + \overleftrightarrow{\mathrm{L}}^a(\mathrm{z}')\delta(\mathrm{z} - \mathrm{z}') \tag{3}$$

with $\overleftrightarrow{g}^a$ the regular part of the Green's dyad and $\overleftrightarrow{L}^a(z)$ the depolarizing dyad contribution, and P.V. indicates a principal

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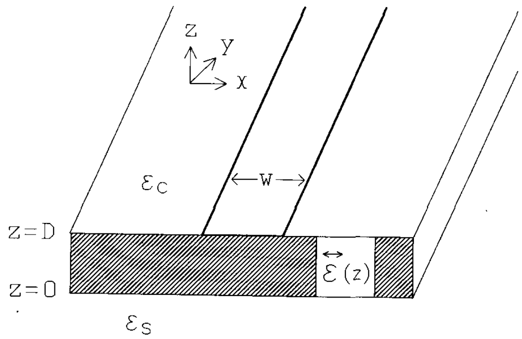

Fig. 1. Microstrip geometry with inhomogeneous anisotropic media.

value type integration on spatial coordinates. Although in the study of infinitely thin microstrip transmission lines the depolarizing dyad contribution is not needed, it is included here for completeness.

If the Green's dyad is known, (1) can be solved by the method of moments (MoM). The unit tangent vector can be decomposed into x and y unit vectors, and the surface currents are expanded with suitable basis functions for the x, y component of current, chosen here as Chebyshev polynomials [9]. Implementing a Galerkin solution by using the same functions for testing as expansion leads to a determinantal eigenvalue equation for complex propagation constant k_y. These modes are found by a numerical root search in the complex k_y-plane. For bound modes, the spectral integration path associated with the Green's function is along the real axis. To detect leaky modes, the path of integration must be deformed correctly around certain branch-point and pole singularities as described in [9]–[12].

To obtain the desired Green's dyad, the microstrip line can be replaced with an arbitrarily located point polarization source ($\vec{P} = \vec{J}/j\omega$). By the equivalence principle, the inhomogeneous anisotropic region can be replaced by a homogeneous isotropic region with equivalent polarization currents which are nonvanishing for $0 \le z \le D$. In the 2-d spectral Fourier transform domain, the unknown equivalent polarization current can be related to an unknown electric field in the region $0 \le z \le D$ by $\vec{P}^{\rm eq}(k_x, k_y, z) = [\overleftrightarrow{\epsilon}(z) - \epsilon_c \overleftrightarrow{I}] \cdot \vec{e}(k_x, k_y, z)$. An IE for the electric field can be formed as

$$\vec{e}(k_x, k_y, z) - jw \int_z \overleftrightarrow{g}_e^i(k_x, k_y, z \mid z') \cdot [\overleftrightarrow{\epsilon}(z') - \epsilon_c \overleftrightarrow{I}] \cdot \vec{e}(k_x, k_y, z')\, dz' = \vec{e}^{\rm inc}(k_x, k_y, z) \quad \cdots 0 \le z \le D \tag{4}$$

where $\overleftrightarrow{g}_e^i$ is an electric dyadic Green's function for the isotropic, homogeneous two region space with the anisotropic material absent, and $\vec{e}^{\rm inc}(k_x, k_y, z)$ is the impressed electric field due to impressed polarization current in the absence of the anisotropic region. The electric dyadic Green's function $\overleftrightarrow{g}_e^i$ can be expressed in a form similar to (3); $\overleftrightarrow{g}_e^i(k_x, k_y, z \mid z') = {\rm PV}\overleftrightarrow{\rm g}^{\rm i}({\rm k_x}, {\rm k_y}, {\rm z} \mid {\rm z'}) + \overleftrightarrow{\rm L}^{\rm i}\delta({\rm z} - {\rm z'})$, where $\overleftrightarrow{L}^i = -\hat{z}\hat{z}/j\omega\epsilon_c$ for a "slice" type principal value. To simplify the notation, (4) can be written as

$$\vec{e}(k_x, k_y, z) - \int_{z'} \overleftrightarrow{g}_e^{\rm ea}(k_x, k_y, z \mid z') \cdot \vec{e}(k_x, k_y, z')\, dz' = \vec{e}^{\rm inc}(k_x, k_y, z) \qquad \cdots 0 \le z \le D \tag{5}$$

where $\overleftrightarrow{g}_e^{\rm ea}(k_x, k_y, z \mid z') = {\rm PV}\ {\rm j}\omega \overleftrightarrow{\rm g}^{\rm ea}({\rm k_x}, {\rm k_y}, {\rm z} \mid {\rm z'}) + \overleftrightarrow{\rm L}^{\rm ea}({\rm z'})\delta({\rm z} - {\rm z'})$ is an "equivalent" electric dyadic Green's function with

$$\begin{aligned} g_{\alpha\alpha}^{\rm ea}(z \mid z') &= g_{\alpha\alpha}^i(z \mid z')[\epsilon_{\alpha\alpha}(z') - \epsilon_c] + g_{\alpha\beta}^i(z \mid z')\epsilon_{\beta\alpha}(z') \\ &\quad + g_{\alpha\gamma}^i(z \mid z')\epsilon_{\gamma\alpha}(z') \quad \alpha \ne \beta \ne \gamma \\ g_{\alpha\beta}^{\rm ea}(z \mid z') &= g_{\alpha\alpha}^i(z \mid z')\epsilon_{\alpha\beta}(z') + g_{\alpha\beta}^i(z \mid z')[\epsilon_{\beta\beta}(z') - \epsilon_c] \\ &\quad + g_{\alpha\gamma}^i(z \mid z')\epsilon_{\gamma\beta}(z') \quad \alpha \ne \beta \ne \gamma \\ \overleftrightarrow{L}^{\rm ea}(z') &= -[\hat{z}\hat{x}\epsilon_{zx}(z') + \hat{z}\hat{y}\epsilon_{zy}(z') + \hat{z}\hat{z}(\epsilon_{zz}(z') - \epsilon_c)]/\epsilon_c. \end{aligned} \tag{6}$$

Due to the nature of the singular behavior of the IE kernel and unknown, (5) should be considered in the distributional sense. The components of the two-layer isotropic Green's dyad are listed in the appendix for convenience.

With the impressed polarization source located at z_0, $\vec{P}^{\rm inc}(k_x, k_y, z) = \hat{\beta}\frac{\delta(z - z_0)}{j\omega}$ the impressed electric field is

$$\vec{e}^{\rm inc}(k_x, k_y, z) = \int_z \overleftrightarrow{g}_e^i(k_x, k_y, z \mid z') \cdot jw\vec{P}^{\rm inc}(k_x, k_y, z')dz' \tag{7}$$

leading to

$$\begin{aligned} &\vec{e}^{\rm inc}(k_x, k_y, z) \\ &\quad = \overleftrightarrow{g}^i(k_x, k_y, z \mid z_0) \cdot \hat{\beta} - \hat{z}(\hat{z} \cdot \hat{\beta})\delta(z - z_0)/j\omega\epsilon_c. \end{aligned} \tag{8}$$

Due to the singular nature of the unknown electric field, the solution of the IE can be decomposed into regular and singular terms as

$$\vec{e}(k_x, k_y, z \mid z_0) = \vec{e}^{\rm reg}(k_x, k_y, z \mid z_0) + \vec{M}_\beta^s(z_0)\delta(z - z_0) \tag{9}$$

where the field/source point dependence $z \mid z_0$ of the solution is included for clarity. Substituting (8) and (9) into (5) and equating regular and singular terms yields

$$\begin{aligned} &\vec{e}^{\rm reg}(k_x, k_y, z \mid z_0) - \int_z \overleftrightarrow{g}_e^{\rm ea}(k_x, k_y, z \mid z') \\ &\quad \cdot \vec{e}^{\rm reg}(k_x, k_y, z' \mid z_0)\, dz' \\ &\quad = \overleftrightarrow{g}^i(k_x, k_y, z \mid z_0) \cdot \hat{\beta} + \overleftrightarrow{g}^{\rm eq}(k_x, k_y, z \mid z_0) \cdot \vec{M}_\beta^s(z_0) \end{aligned} \tag{10a}$$

$$\vec{M}_\beta^s(z_0) = -\frac{\hat{z}(\hat{z} \cdot \hat{\beta})}{j\omega\epsilon_c} + \overleftrightarrow{L}^{\rm ea}(z_0) \cdot \vec{M}_\beta^s(z_0) \tag{10b}$$

where (10b) and the last term in (10a) vanish unless $0 \le z_0 \le D$. Equation (10b) can be solved analytically, resulting in

$$\vec{M}_\beta^s(z_0) = -\frac{\hat{z}(\hat{z} \cdot \hat{\beta})}{j\omega\epsilon_{zz}(z_0)} \tag{11}$$

and (10a) is solved numerically for the regular part of the unknown electric field.

The desired Green's function can be constructed from the regular and singular parts of the solution of (10). For field points $0 \leq z \leq D$, the solution provides the field at z due to a point source at z_0, therefore the field is by definition the desired Green's function. For $z > D$, the Green's function is the sum of the field due to the equivalent polarization currents in the region $0 \leq z \leq D$ and the direct field of the incident polarization current radiating in the space $z > D$. The Green's dyad has the form of (3), with components identified as (12), shown at the bottom of the page. The depolarizing dyad term for $0 < z < D$ is identical to that obtained in [13] for similar media.

III. Numerical Solution and Results

IE (10a) is solved using a pulse-function, MoM/Galerkin procedure. The unknown field over the range $0 \leq z \leq D$ is expanded in a set of pulse functions as

$$\vec{e}^{\text{reg}}(k_x, k_y, z) = \sum_{\beta=x,y,z} \sum_{n=1}^{N} \hat{\beta} a_n^{\beta} p_n(z) \tag{13}$$

where $p_n(z) = \begin{cases} 1 & z_n - \frac{w_n}{2} \leq z \leq z_n + \frac{w_n}{2} \\ 0 & \text{otherwise} \end{cases}$ with w_n the width of the nth pulse. Testing with $\int_z dz\, p_{ng}(z)\hat{\alpha}$ results in a $(3\ N) \times (\ 3N)$ matrix system which can be solved for the unknown amplitudes a_n^{β}. The spatial integrals associated with expansion and testing can be performed in closed form, so that the MoM matrix entries are determined analytically.

It should be noted that further subdividing any anisotropic region into more layers does not complicate the solution, since the appropriate material parameter can be assigned to each subdomain expansion function for stratified media. For continuously varying permittivity distributions the correct functional dependence of the permittivity can be incorporated into the spatial integrals, eliminating the need to approximate the region be many thin, constant-permittivity layers.

In order to verify the IE method presented here, Fig. 2 shows various Green's function components for the special case of a grounded uniaxial slab. The dominant component, either real or imaginary, is shown. The components g_{xx}, g_{xy}, g_{yy} are compared to an analytical formulation [14], and agreement is seen to be very good. Other checks performed but not shown include a comparison to all nine analytically determined components for an isotropic, three-layer geometry [15], and an isotropic grounded slab geometry with suspended cover [16]. The percent error between the analytical function and the numerical solution of (10) for all cases was typically much less than 0.1% using 20 pulses. In addition, due to the smooth nature of the functions except near the origin, each function was pre-computed at several values of k_x, and interpolated for efficiency.

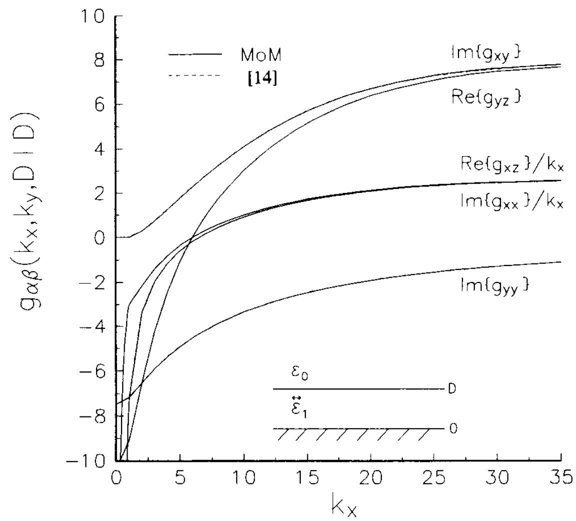

Fig. 2. Various Green's dyad components versus k_x for k_y = (3,0), D = 0.0635 cm, $\epsilon_{xx} = \epsilon_{yy} = 9.4$, $\epsilon_{zz} = 11.6$ (sapphire). MoM: —, analytical results for g_{xx}, g_{xy}, g_{yy} from [14]: ----.

To validate the microstrip solution using the new Green's dyad formulation, a comparison with previously published results was made for microstrip dispersion characteristics of the dominant and higher order modes. Results were compared with lines on uniaxial substrates for the dominant and first higher order mode in the bound regime [17], the dominant mode on uniaxial substrates for several orientations of the optic axis [18], the dominant mode on biaxial substrates [19], and higher order modes in the leaky regime for isotropic substrates [9]. In all cases this method was found to be in excellent agreement with previous results.

Fig. 3 shows results for a microstrip transmission line on a grounded anisotropic substrate (sapphire) for various orientations of the optic axis, where θ, ϕ are the usual spherical angles. This geometry is similar to that studied in [9], [11] for an isotropic substrate ($\epsilon_r = 9.8$). The dominant and first higher-order mode are shown. The lowest order surface-wave mode (SW_0) is also shown, which is TM when the principle crystal axis are aligned with the geometrical axis, and hybrid otherwise. This mode is calculated from (5) under sourceless conditions, with $k_x = 0$, where a root search yields the surface-wave propagation constant. When the higher-order mode phase constant falls below k_0 and SW_0, the mode leaks into space and surface waves. It is seen that the propagation constant is only weakly dependent on anisotropy in the leaky regime, since the mode is loosely confined to the substrate near cutoff. In the bound regime, and for increasing frequency, the mode

$$\overleftrightarrow{g}^{a}(k_x, k_y, z \mid z_0) \cdot \hat{\beta} = \begin{cases} \vec{e}^{\text{reg}}(k_x, k_y, z \mid z_0) & 0 < z < D \\ \overleftrightarrow{g}^{i}(k_x, k_y, z \mid z_0) \cdot \hat{\beta} + \int_z \overleftrightarrow{g}_e^{\text{ea}}(k_x, k_y, z \mid z') \cdot \vec{e}^{\text{reg}}(k_x, k_y, z' \mid z_0)\, dz' & z > D \end{cases}$$
$$\overleftrightarrow{L}^{a}(z_0) = \begin{cases} -\frac{\hat{z}\hat{z}}{j\omega\epsilon_{zz}(z_0)} & 0 < z < D \\ -\frac{\hat{z}\hat{z}}{j\omega\epsilon_c} & z > D \end{cases} \tag{12}$$

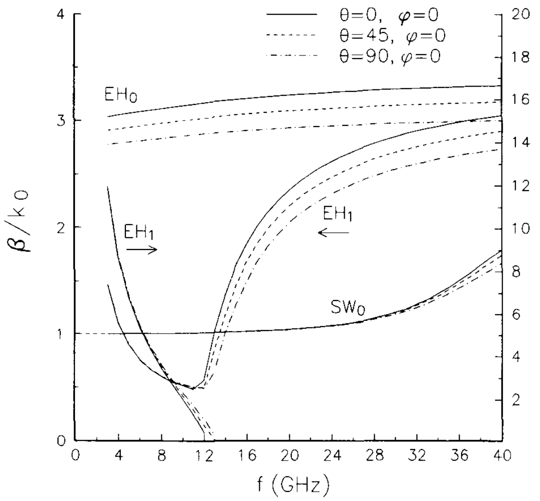

Fig. 3. Dispersion curves for the fundamental and first higher order mode for microstrip on sapphire, for various orientations of the optic axis in the plane normal to propagation. $D = 0.0635$ cm, $W = 0.3$ cm.

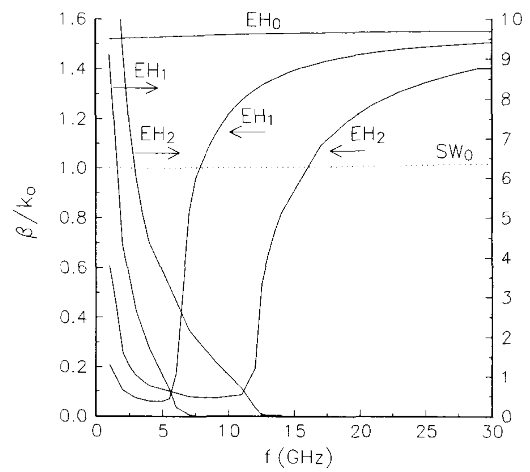

Fig. 4. Dispersion curves for the fundamental and first two higher order modes for microstrip on biaxial PTFE substrate,with permittivity values from [19]. $D = 0.05$ cm, $W = 1.5$ cm, $\epsilon_{xx} = 2.89$, $\epsilon_{yy} = 2.95$, $\epsilon_{zz} = 2.45$.

becomes more tightly confined to the substrate, and anisotropy of the substrate becomes important.

Fig. 4 shows the dominant and first two higher-order modes for microstrip on a biaxial PTFE substrate with $\epsilon_{xx} = 2.89$, $\epsilon_{yy} = 2.95$, $\epsilon_{zz} = 2.45$ [19], $D = 0.05$ cm, $W = 1.5$ cm. This geometry was studied in [9], [11], [20], again for an isotropic substrate ($\epsilon_r = 2.32$, $D = .0794$) . These results are similar to the isotropic case in the leaky regime, which is expected for loosely bound modes.

IV. Conclusion

A straightforward numerical technique to determine the complete spectral domain Green's dyad for inhomogeneous anisotropic media has been presented. This method is relevant to guided-wave problems such as transmission lines embedded in anisotropic media, where propagation characteristics are desired in the axial transform domain. Spectral Green's components are determined from a one-dimensional method of moments solution to a polarization-type integral equation. This method is very simple and versatile, and can be used to model continuously varying or stratified dielectric media with permittivity dyads of the most general form. An application to microstrip transmission line propagation on anisotropic substrates was shown, and results for propagation and leakage were obtained.

Appendix

The Green's dyad components for an isotropic two-layer geometry in the two-dimensional Fourier transform plane (k_x, k_y) for any $(z, z' \geq 0)$ are given as

$$g^i_{xx} = \{(k_c^2 - k_x^2)\left[e^{-p_c|z-z'|} + R_t e^{-p_c(z+z')}\right] + k_x^2 p_c C e^{-p_c(z+z')}\}/2p_c j\omega\epsilon_c$$

$$g^i_{xy} = \{k_x k_y (p_c C - R_t) e^{-p_c(z+z')} - k_x k_y e^{-p_c|z-z'|}\}/2p_c j\omega\epsilon_c$$

$$g^i_{xz} = \{-jk_x p_c\left[\mathrm{sgn}(z-z')e^{-p_c|z-z'|} + R_n e^{-p_c(z+z')}\right]\}/2p_c j\omega\epsilon_c$$

$$g^i_{yx} = g^i_{xy}$$

$$g^i_{yy} = \{(k_c^2 - k_y^2)\left[e^{-p_c|z-z'|} + R_t e^{-p_c(z+z')}\right] + k_y^2 p_c C e^{-p_c(z+z')}\}/2p_c j\omega\epsilon_c$$

$$g^i_{yz} = \{-jk_y p_c\left[\mathrm{sgn}(z-z')e^{-p_c|z-z'|} + R_n e^{-p_c(z+z')}\right]\}/2p_c j\omega\epsilon_c$$

$$g_{zx} = \{(k_c^2 + p_c^2) jk_x C e^{-p_c(z+z')} - jk_x p_c \times \left[\mathrm{sgn}(z-z')e^{-p_c|z-z'|} + R_t e^{-p_c(z+z')}\right]\}/2p_c j\omega\epsilon_c$$

$$g^i_{zy} = \{(k_c^2 + p_c^2) jk_y C e^{-p_c(z+z')} - jk_y p_c \times \left[\mathrm{sgn}(z-z')e^{-p_c|z-z'|} + R_t e^{-p_c(z+z')}\right]\}/2p_c j\omega\epsilon_c$$

$$g^i_{zz} = \{(k_c^2 + p_c^2)\left[e^{-p_c|z-z'|} + R_n e^{-p_c(z+z')}\right]\}/2p_c j\omega\epsilon_c$$

where

$$R_t = \frac{p_c - p_s}{p_c + p_s}, \quad R_n = \frac{N^2 p_c - p_s}{N^2 p_c + p_s},$$

$$C = \frac{2(N^2 - 1)p_c}{(p_c + p_s)(N^2 p_c + p_s)}$$

with $p_{c,s} = \sqrt{k_x^2 + k_y^2 - k_{c,s}^2}, N = n_s/n_c, n_{s,c}\ k_c = n_c k_0$ and $\mathrm{sgn}(z) = \begin{cases} 1 & z > 0 \\ -1 & z < 0 \\ 0 & z = 0 \end{cases}.$

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