A Numerical Formulation of Dyadic Green's Functions for Planar Bianisotropic Media with Application to Printed Transmission Lines

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Abstract—**An integral equation (IE) method with numerical solution is presented to determine the complete Green's dyadic for planar bianisotropic media. This method follows directly from the linearity of Maxwell's equations upon applying the volume equivalence principle for general linear media. The Green's function components are determined by the solution of two coupled one-dimensional IE's, with the regular part determined numerically and the depolarizing dyad contribution determined analytically. This method is appropriate for generating Green's functions for the computation of guided-wave propagation characteristics of conducting transmission lines and dielectric waveguides. The formulation is relatively simple, with the kernels of the IE's to be solved involving only linear combinations of Green's functions for an isotropic half-space. This method is verified by examining various results for microstrip transmission lines with electrically and magnetically anisotropic substrates, nonreciprocal ferrite superstrates, and chiral substrates. New results are presented for microstrip embedded in chiroferrite media.**

I. Introduction

TRANSMISSION LINES and waveguiding structures implemented in planar geometries are important components of hybrid and monolithic integrated circuits. As material processing and fabrication techniques have matured, novel circuit structures have been implemented by controlling the electromagnetic properties of the circuit materials, often through the use of naturally occurring or intentionally induced anisotropy. For example, biased ferrites, which have anisotropic permeability, have been used in various nonreciprocal devices such as phase shifters and isolators, where dynamic control over performance characteristics can be achieved by varying the applied magnetic bias field [1], [2]. Similar comments apply to solid-state magnetoplasmas in [3], which have anisotropic permittivity, and are most appropriate for millimeter-wave devices. Chiral media, which are usually bi-isotropic, have been investigated for antenna radomes, as polarization transformers, and as microstrip circuit materials, among other uses [4]–[10]. Other complex types of media, such as anisotropic chiral materials in [11], and combinations of chiral materials and ferrites (chiroferrites) in [12] and [13] are also being studied. The above few references are intended to provide only representative examples of past research, since a large amount of work has been done in this area.

In order to determine the propagation characteristics of transmission lines and waveguides embedded in general linear media, integral equation (IE) techniques are often used. The IE method provides physical insight, yields accurate results, and is usually straightforward once the Green's function for the surrounding environment is determined. Analytical determination of the Green's function for multilayered bianisotropic media is extremely complicated, and often seminumerical techniques are employed. Several representative references for complex-media Green's functions are made here. The dyadic Green's function for multilayered electrically anisotropic structures is considered in [14] and [15], and electrically and magnetically anisotropic media are considered in [1], [2], and [16]–[18], with some numerical applications to antennas and transmission lines. The Green's dyadic for fully bianisotropic media is considered in [19], and with waveguiding applications in [20] and [21].

A relatively simple numerical method for determining the complete Green's dyadic for general linear, planar media is presented here. This method accommodates multiple layers easily, allows for a full complex-valued matrix representation of the permittivity, permeability, and optical activity dyadics as continuous functions of position normal to the planar layering, and yields results that are analytically integrable in directions transverse to the planar layering. The formulation is relatively simple, with the kernels of the IE's to be solved involving only linear combinations of Green's functions for an isotropic half-space. The method presented here is an extension of previous work on inhomogeneous electrically anisotropic media [22]. This method is verified by comparing results with those previously published for a microstrip transmission line printed on an electrically and magnetically anisotropic substrate, a chiral substrate, and for a microstrip over an isotropic substrate with a biased ferrite superstrate. New results are presented for a microstrip embedded in chiroferrite media. The method is general and applicable to one-, two-, and three-dimensional problems, although three-dimensional applications such as the study of end effects or discontinuities would probably require a more efficient solution of the defining IE's than the present scheme described here.

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II. THEORY

Maxwell's equations in the frequency domain ($e^{j\omega t}$ assumed and suppressed) can be written as

$$\begin{aligned} \nabla \times \vec{E} &= -j\omega\vec{B} - \vec{J}_m \\ \nabla \times \vec{H} &= j\omega\vec{D} + \vec{J}_e \end{aligned} \qquad (1)$$

where $\vec{J}_{e/m}$ are electric/magnetic currents, with the constitutive relations for bianisotropic media given by

$$\begin{aligned} \vec{D} &= \overleftrightarrow{\epsilon} \cdot \vec{E} + \overleftrightarrow{\xi} \cdot \vec{H} \\ \vec{B} &= \overleftrightarrow{\mu} \cdot \vec{H} + \overleftrightarrow{\zeta} \cdot \vec{E}. \end{aligned} \qquad (2)$$

Because of the linearity of (1) and (2), the following relations between fields and currents hold [23]

$$\begin{aligned} \vec{E}(\vec{r}) &= \int_V \overleftrightarrow{G}_{bi}^{e,e}(\vec{r}|\vec{r}\,') \cdot \vec{J}_e(\vec{r}\,')\,dV' \\ &\quad + \int_V \overleftrightarrow{G}_{bi}^{e,h}(\vec{r}|\vec{r}\,') \cdot \vec{J}_m(\vec{r}\,')\,dV' \\ \vec{H}(\vec{r}) &= \int_V \overleftrightarrow{G}_{bi}^{h,e}(\vec{r}|\vec{r}\,') \cdot \vec{J}_e(\vec{r}\,')\,dV' \\ &\quad + \int_V \overleftrightarrow{G}_{bi}^{h,h}(\vec{r}|\vec{r}\,') \cdot \vec{J}_m(\vec{r}\,')\,dV' \end{aligned} \qquad (3)$$

where $\overleftrightarrow{G}_{bi}^{\alpha,\beta}$ is the dyadic Green's function which provides the vector field of type α due to the vector current of type β in the bianisotropic medium. Each dyadic Green's function can be decomposed into a regular part and a depolarizing dyadic contribution [24], [25]

$$\overleftrightarrow{G}_{bi}^{\alpha,\beta}(\vec{r}|\vec{r}\,') = P.V.\ \overleftrightarrow{G}_{bi,r}^{\alpha,\beta}(\vec{r}|\vec{r}\,') + \overleftrightarrow{L}^{\alpha,\beta}(z')\delta(\vec{r} - \vec{r}\,') \qquad (4)$$

where P.V. indicates a principal value type integration in (3) to avoid the point $\vec{r} = \vec{r}\,'$. The regular part can be written in a plane-wave spectral form

$$\begin{aligned} \overleftrightarrow{G}_{bi,r}^{\alpha,\beta}(\vec{r}|\vec{r}\,') &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \overleftrightarrow{g}_{bi,r}^{\alpha,\beta}(k_x, k_y, z|z') \\ &\quad \cdot e^{jk_x(x-x')}e^{jk_y(y-y')}\,dk_x\,dk_y. \end{aligned} \qquad (5)$$

The Green's dyadics are completely determined by specifying the depolarizing dyadics $\overleftrightarrow{L}^{\alpha,\beta}(z')$ and the spectral terms $\overleftrightarrow{g}_{bi,r}^{\alpha,\beta}(k_x, k_y, z|z')$, which can be obtained by the solution of a spatially one-dimensional problem. In the following, the depolarizing terms $\overleftrightarrow{L}^{\alpha,\beta}(z')$ are determined analytically (for a "slice" principal volume [25]), and the spectral domain Green's dyadics $\overleftrightarrow{g}_{bi,r}^{\alpha,\beta}(k_x, k_y, z|z')$ are determined numerically by solving a one-dimensional set of coupled polarization-type IE's. The remainder of this paper will concentrate on determining the above quantities, and finally applications will be presented for microstrip transmission lines in complex media, as shown in Fig. 1.

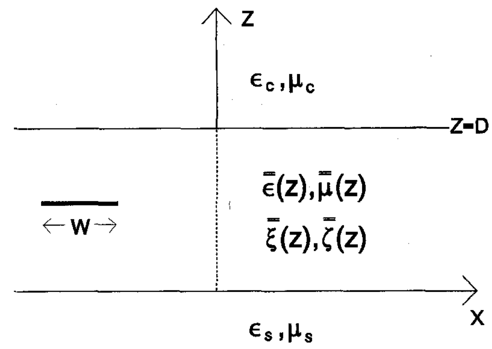

Fig. 1. A microstrip transmission line embedded in a planar bianisotropic medium.

The formulation of IE's to obtain the desired Green's functions proceeds as follows. Invoking the transform pair

$$\begin{aligned} \vec{f}(k_x, k_y, z) &= \iint_{-\infty}^{\infty} \vec{F}(x, y, z)e^{-jk_x x}e^{-jk_y y}\,dx\,dy \\ \vec{F}(x, y, z) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \vec{f}(k_x, k_y, z)e^{jk_x x} \cdot e^{jk_y y}\,dk_x\,dk_y \end{aligned} \qquad (6)$$

allows a set of IE's to be formed directly in the (k_x, k_y) plane. The spectral representation of a point source of electric or magnetic type is applied to the inhomogeneous structure as shown in Fig. 2(a), resulting in a set of unknown fields $(\vec{e}, \vec{h})$. The inhomogeneous bianisotropic medium along with the unknown fields can be replaced with a homogeneous isotropic medium and unknown polarization currents as shown in Fig. 2(b), via the volume equivalence theorem for bianisotropic media. The volume equivalence theorem for chiral isotropic media is presented in [8]. The generalization to bianisotropic media is a simple extension of the above specialized case, and so the derivation is not presented here, but results in

$$\begin{aligned} \vec{J}_e^{eq} &= j\omega[\overleftrightarrow{\epsilon}(z') - \epsilon_c\overleftrightarrow{I}] \cdot \vec{e} + j\omega\overleftrightarrow{\xi}(z') \cdot \vec{h} \\ \vec{J}_m^{eq} &= j\omega[\overleftrightarrow{\mu}(z') - \mu_c\overleftrightarrow{I}] \cdot \vec{h} + j\omega\overleftrightarrow{\zeta}(z') \cdot \vec{e} \end{aligned} \qquad (7)$$

where the dependence of the field and current vectors on (k_x, k_y) is suppressed above and in the following.

A set of IE's can be formed for the unknown polarization currents immersed in the homogeneous isotropic region $z > 0$ by forcing the total field to equal the impressed field due to the impressed current plus the scattered field due to the polarization currents, $\vec{e} = \vec{e}^{\,i} + \vec{e}^{\,s}$, $\vec{h} = \vec{h}^i + \vec{h}^s$. Inserting transform-domain relations similar to (3), but for the situation depicted in Fig. 2(b), into the above condition results in

$$\begin{aligned} \vec{e}(z) &- \int_z \overleftrightarrow{g}^{e,e}(z|z') \cdot \vec{J}_e^{eq}(z')\,dz' \\ &- \int_z \overleftrightarrow{g}^{e,h}(z|z') \cdot \vec{J}_m^{eq}(z')\,dz' = \vec{e}^{\,i}(z) \\ \vec{h}(z) &- \int_z \overleftrightarrow{g}^{h,e}(z|z') \cdot \vec{J}_e^{eq}(z')\,dz' \\ &- \int_z \overleftrightarrow{g}^{h,h}(z|z') \cdot \vec{J}_m^{eq}(z')\,dz' = \vec{h}^i(z) \\ &\cdots 0 < z < D \end{aligned} \qquad (8)$$

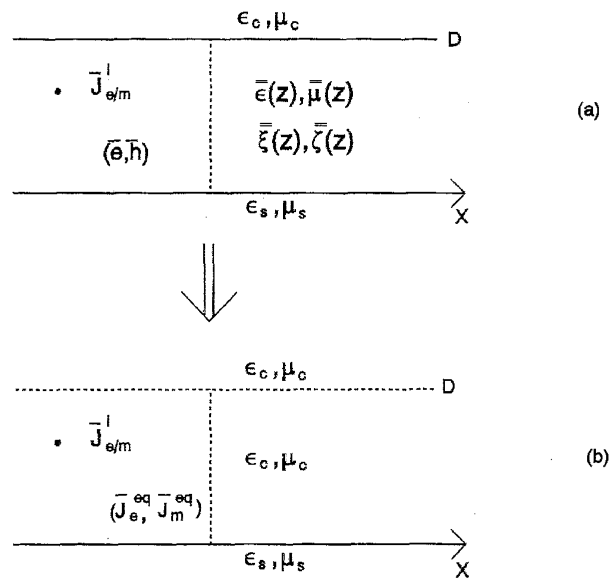

Fig. 2. (a) Electric or magnetic current source embedded in a planar bianisotropic slab, producing unknown fields $(\vec{e}, \vec{h})$. (b) Structure equivalent to (a) via the bianisotropic volume equivalence principle. Inhomogeneous bianisotropic medium with unknown fields $(\vec{e}, \vec{h})$ is replaced with homogeneous isotropic medium with unknown polarization currents $(\vec{J}_e^{eq}, \vec{J}_m^{eq})$.

where the incident fields are given by

$$\begin{aligned}\vec{e}^{\,i}(z) &= \int_z \overleftrightarrow{g}^{e,e}(z|z')\cdot\vec{J}_e^{\,i}(z')\,dz' \\ &\quad + \int_z \overleftrightarrow{g}^{e,h}(z|z')\cdot\vec{J}_m^{\,i}(z')\,dz' \\ \vec{h}^{i}(z) &= \int_z \overleftrightarrow{g}^{h,e}(z|z')\cdot\vec{J}_e^{\,i}(z')\,dz' \\ &\quad + \int_z \overleftrightarrow{g}^{h,h}(z|z')\cdot\vec{J}_m^{\,i}(z')\,dz'.\end{aligned} \tag{9}$$

In (8) and (9), the Green's functions $\overleftrightarrow{g}^{\alpha,\beta}$ are for the simple isotropic two-region geometry shown in Fig. 2(b), having (ϵ_c, μ_c) for $z>0$ and (ϵ_s, μ_s) for $z<0$, with the bianisotropic medium absent. These functions are provided in the Appendix for convenience. Upon inserting (7) into (8) the IE's can be written in a simple form as

$$\begin{aligned}\vec{e}(z) &- \int_z \overleftrightarrow{g}^{e,eq}(z|z')\cdot\vec{e}(z')\,dz' \\ &- \int_z \overleftrightarrow{g}^{e,hq}(z|z')\cdot\vec{h}(z')\,dz' = \vec{e}^{\,i}(z) \\ \vec{h}(z) &- \int_z \overleftrightarrow{g}^{h,eq}(z|z')\cdot\vec{e}(z')\,dz' \\ &- \int_z \overleftrightarrow{g}^{h,hq}(z|z')\cdot\vec{h}(z')\,dz' = \vec{h}^{i}(z) \\ &\cdots 0<z<D\end{aligned} \tag{10}$$

where

$$\begin{aligned}\overleftrightarrow{g}^{e,eq}(z|z') &= j\omega\overleftrightarrow{g}^{e,e}(z|z')\cdot[\overleftrightarrow{\epsilon}(z')-\epsilon_c\overleftrightarrow{I}] \\ &\quad + j\omega\overleftrightarrow{g}^{e,h}(z|z')\cdot\overleftrightarrow{\zeta}(z') \\ \overleftrightarrow{g}^{e,hq}(z|z') &= j\omega\overleftrightarrow{g}^{e,e}(z|z')\cdot\overleftrightarrow{\xi}(z') \\ &\quad + j\omega\overleftrightarrow{g}^{e,h}(z|z')\cdot[\overleftrightarrow{\mu}(z')-\mu_c\overleftrightarrow{I}] \\ \overleftrightarrow{g}^{h,eq}(z|z') &= j\omega\overleftrightarrow{g}^{h,h}(z|z')\cdot\zeta(z') \\ &\quad + j\omega\overleftrightarrow{g}^{h,e}(z|z')\cdot[\overleftrightarrow{\epsilon}(z')-\epsilon_c\overleftrightarrow{I}] \\ \overleftrightarrow{g}^{h,hq}(z|z') &= j\omega\overleftrightarrow{g}^{h,h}(z|z')\cdot[\overleftrightarrow{\mu}(z')-\mu_c\overleftrightarrow{I}] \\ &\quad + j\omega\overleftrightarrow{g}^{h,e}(z|z')\cdot\overleftrightarrow{\xi}(z').\end{aligned} \tag{11}$$

It should be noted that (10) would have the same form for a simple isotropic medium characterized by (ϵ, μ), with the only difference being that the Green's functions used in (10) are combinations of Green's functions for both source types (electric and magnetic), rather than for a single source type as when $\overleftrightarrow{\xi} = \overleftrightarrow{\zeta} = 0$. This means, for instance, that a program written to analyze scattering from media characterized by (ϵ, μ) by a volume formulation can be extended to analyze media characterized by $(\overleftrightarrow{\epsilon}, \overleftrightarrow{\mu}, \overleftrightarrow{\xi}, \overleftrightarrow{\zeta})$ by simply replacing the isotropic-media Green's function with an appropriate combination of isotropic-media Green's functions (i.e., (11)), at no additional computational expense and minimal added complexity.

If the impressed sources are located at z_0 and are given by $\vec{J}_e^{\,i} = \hat{\beta}_e^0\delta(z-z_0)J_e^0$ for the electric type and $\vec{J}_m^{\,i} = \hat{\beta}_m^0\delta(z-z_0)J_m^0$ for the magnetic type, insertion into (10) yields

$$\begin{aligned}\vec{e}(z) &- \int_z \overleftrightarrow{g}^{e,eq}(z|z')\cdot\vec{e}(z')\,dz' \\ &- \int_z \overleftrightarrow{g}^{e,hq}(z|z')\cdot\vec{h}(z')\,dz' \\ &= \left[\overleftrightarrow{g}_r^{e,e}(z|z_0) - \frac{\hat{z}\hat{z}\delta(z-z_0)}{j\omega\epsilon_c}\right]\cdot\hat{\beta}_e^0 J_e^0 \\ &\quad + \overleftrightarrow{g}^{e,h}(z|z_0)\cdot\hat{\beta}_m^0 J_m^0\end{aligned} \tag{12a}$$

$$\begin{aligned}\vec{h}(z) &- \int_z \overleftrightarrow{g}^{h,eq}(z|z') \cdot \vec{e}(z')\,dz' \\ &- \int_z \overleftrightarrow{g}^{h,hq}(z|z') \cdot \vec{h}(z')\,dz' \\ &= \left[\overleftrightarrow{g}^{h,e}(z|z_0) - \frac{\hat{z}\hat{z}\delta(z-z_0)}{j\omega\mu_c}\right] \cdot \hat{\beta}_e^0 J_e^0 \\ &+ \overleftrightarrow{g}_r^{h,h}(z|z_0) \cdot \hat{\beta}_m^0 J_m^0 \\ &\cdots 0 < z < D. \end{aligned} \tag{12b}$$

Due to the expected singular nature of the unknown fields if $0 < z_0 < D$, the solution of the IE's can be decomposed into regular and singular terms [22] as

$$\begin{aligned}\vec{e}(z|z_0) &= \vec{e}_r(z|z_0) + \vec{M}_{\beta_e^0}^e(z_0)\delta(z-z_0) \\ \vec{h}(z|z_0) &= \vec{h}_r(z|z_0) + \vec{M}_{\beta_h^0}^h(z_0)\delta(z-z_0)\end{aligned} \tag{13}$$

where the field/source point dependence $z|z_0$ of the solution is included for clarity. Substituting (13) into (12) and equating regular and singular terms yields

$$\begin{aligned}\vec{e}_r(z|z_0) &- \int_z \overleftrightarrow{g}^{e,eq}(z|z') \cdot \vec{e}_r(z'|z_0)\,dz' \\ &- \overleftrightarrow{g}_r^{e,eq}(z|z_0) \cdot \vec{M}_{\beta_e^0}^e(z_0)\delta_D \\ &- \int_z \overleftrightarrow{g}^{e,hq}(z|z') \cdot \vec{h}_r(z'|z_0)\,dz' \\ &- \overleftrightarrow{g}_r^{e,hq}(z|z_0) \cdot \vec{M}_{\beta_h^0}^h(z_0)\delta_D \\ &= \overleftrightarrow{g}_r^{e,e}(z|z_0) \cdot \hat{\beta}_e^0 J_e^0 + \overleftrightarrow{g}^{e,h}(z|z_0) \cdot \hat{\beta}_m^0 J_m^0 \\ &\cdots 0 < z < D \end{aligned} \tag{14a}$$

$$\begin{aligned}\vec{M}_{\beta_e^0}^e(z_0) &- \overleftrightarrow{L}^{e,eq}(z_0) \cdot \vec{M}_{\beta_e^0}^e(z_0) - \overleftrightarrow{L}^{e,ha} \cdot \vec{M}_{\beta_h^0}^h(z_0) \\ &= \frac{-\hat{z}(\hat{z}\cdot\hat{\beta}_e^0)J_e^0}{j\omega\epsilon_c} \quad \cdots 0 < z_0 < D\end{aligned} \tag{14b}$$

from (12a) and

$$\begin{aligned}\vec{h}_r(z|z_0) &- \int_z \overleftrightarrow{g}^{h,eq}(z|z') \cdot \vec{e}_r(z'|z_0)\,dz' \\ &- \overleftrightarrow{g}_r^{h,eq}(z|z_0) \cdot \vec{M}_{\beta_e^0}^e(z_0)\delta_D \\ &- \int_z \overleftrightarrow{g}^{h,hq}(z|z') \cdot \vec{h}_r(z'|z_0)\,dz' \\ &- \overleftrightarrow{g}_r^{h,hq}(z|z_0) \cdot \vec{M}_{\beta_h^0}^h(z_0)\delta_D \\ &= \overleftrightarrow{g}_r^{h,e}(z|z_0) \cdot \hat{\beta}_e^0 J_e^0 + \overleftrightarrow{g}^{h,h}(z|z_0) \cdot \hat{\beta}_m^0 J_m^0 \\ &\cdots 0 < z < D \end{aligned} \tag{15a}$$

$$\begin{aligned}\vec{M}_{\beta_h^0}^h(z_0) &- \overleftrightarrow{L}^{h,eq}(z_0) \cdot \vec{M}_{\beta_e^0}^e(z_0) - \overleftrightarrow{L}^{h,ha} \cdot \vec{M}_{\beta_h^0}^h(z_0) \\ &= \frac{-\hat{z}(\hat{z}\cdot\hat{\beta}_m^0)J_m^0}{j\omega\mu_c} \quad \cdots 0 < z_0 < D\end{aligned} \tag{15b}$$

from (12b), where

$$\delta_D = \begin{cases} 1 & 0 < z_0 < D \\ 0 & \text{otherwise.} \end{cases}$$

Equations (14b) and (15b) are two coupled equations for the singular part of the solution of $(\vec{e}, \vec{h})$, which can be solved analytically to yield

$$\begin{aligned}\vec{M}_{\beta^0}^e(z_0) &= \frac{-\hat{z}(\hat{z}\cdot\hat{\beta}^0)[\mu_{zz}(z_0)J_e^0 - \xi_{zz}(z_0)J_m^0]}{j\omega[\epsilon_{zz}(z_0)\mu_{zz}(z_0) - \zeta_{zz}(z_0)\xi_{zz}(z_0)]} \\ \vec{M}_{\beta^0}^h(z_0) &= \frac{-\hat{z}(\hat{z}\cdot\hat{\beta}^0)[\epsilon_{zz}(z_0)J_m^0 - \zeta_{zz}(z_0)J_e^0]}{j\omega[\epsilon_{zz}(z_0)\mu_{zz}(z_0) - \zeta_{zz}(z_0)\xi_{zz}(z_0)]}\end{aligned} \tag{16}$$

where $\hat{\beta}_e^0 = \hat{\beta}_h^0 = \hat{\beta}^0$ for convenience.

The desired Green's function can be constructed from the regular and singular parts of the solution of (14) and (15). For field points $0 \le z \le D$, the solution provides the field at z due to a point source at z_0, therefore, the field is by definition the desired Green's function in the (k_x, k_y, z) plane. For an electric source ($J_e^0 = 1$, $J_m^0 = 0$),

$$\begin{aligned}\overleftrightarrow{g}_{bi,r}^{e,e}(z|z_0) \cdot \hat{\beta}^0 &= \vec{e}_r(z|z_0) \\ \overleftrightarrow{g}_{bi,r}^{h,e}(z|z_0) \cdot \hat{\beta}^0 &= \vec{h}_r(z|z_0)\end{aligned}$$

$$\begin{aligned}\overleftrightarrow{L}_{bi}^{e,e}(z_0) &= \frac{-\hat{z}\hat{z}\mu_{zz}(z_0)}{j\omega[\epsilon_{zz}(z_0)\mu_{zz}(z_0) - \zeta_{zz}(z_0)\xi_{zz}(z_0)]} \\ \overleftrightarrow{L}_{bi}^{h,e}(z_0) &= \frac{\hat{z}\hat{z}\zeta_{zz}(z_0)}{j\omega[\epsilon_{zz}(z_0)\mu_{zz}(z_0) - \zeta_{zz}(z_0)\xi_{zz}(z_0)]}\end{aligned} \tag{17}$$

whereas, for a magnetic source ($J_e^0 = 0$, $J_m^0 = 1$)

$$\begin{aligned}\overleftrightarrow{g}_{bi,r}^{e,h}(z|z_0) \cdot \hat{\beta}^0 &= \vec{e}_r(z|z_0) \\ \overleftrightarrow{g}_{bi,r}^{h,h}(z|z_0) \cdot \hat{\beta}^0 &= \vec{h}_r(z|z_0)\end{aligned}$$

$$\begin{aligned}\overleftrightarrow{L}_{bi}^{e,h}(z_0) &= \frac{\hat{z}\hat{z}\xi_{zz}(z_0)}{j\omega[\epsilon_{zz}(z_0)\mu_{zz}(z_0) - \zeta_{zz}(z_0)\xi_{zz}(z_0)]} \\ \overleftrightarrow{L}_{bi}^{h,h}(z_0) &= \frac{-\hat{z}\hat{z}\epsilon_{zz}(z_0)}{j\omega[\epsilon_{zz}(z_0)\mu_{zz}(z_0) - \zeta_{zz}(z_0)\xi_{zz}(z_0)]}.\end{aligned} \tag{18}$$

For $z > D$, the desired Green's function is the sum of the field due to the equivalent polarization currents radiating into the isotropic half-space and the direct field of the impressed source. For an electric source

$$\begin{aligned}\overleftrightarrow{g}_{bi}^{e,e}(z|z_0) \cdot \hat{\beta}^0 &= \int_z \overleftrightarrow{g}^{e,eq}(z|z') \cdot \vec{e}_r(z'|z_0)\,dz' \\ &+ \int_z \overleftrightarrow{g}^{e,hq}(z|z') \cdot \vec{h}_r(z'|z_0)\,dz' \\ &+ \overleftrightarrow{g}^{e,e}(z|z_0) \cdot \hat{\beta}^0 \\ \overleftrightarrow{g}_{bi}^{h,e}(z|z_0) \cdot \hat{\beta}^0 &= \int_z \overleftrightarrow{g}^{h,eq}(z|z') \cdot \vec{e}_r(z'|z_0)\,dz' \\ &+ \int_z \overleftrightarrow{g}^{h,hq}(z|z') \cdot \vec{h}_r(z'|z_0)\,dz' \\ &+ \overleftrightarrow{g}^{h,e}(z|z_0) \cdot \hat{\beta}^0\end{aligned} \tag{19}$$

and for a magnetic source

$$\begin{aligned}\overleftrightarrow{g}_{bi}^{e,h}(z|z_0) \cdot \hat{\beta}^0 &= \int_z \overleftrightarrow{g}^{e,eq}(z|z') \cdot \vec{e}_r(z'|z_0)\,dz' \\ &+ \int_z \overleftrightarrow{g}^{e,hq}(z|z') \cdot \vec{h}_r(z'|z_0)\,dz' \\ &+ \overleftrightarrow{g}^{e,h}(z|z_0) \cdot \hat{\beta}^0\end{aligned}$$

$$\overleftrightarrow{g}_{bi}^{h,h}(z|z_0)\cdot\hat{\beta}^0 = \int_z \overleftrightarrow{g}^{h,eq}(z|z')\cdot\vec{e}_r(z'|z_0)\,dz' + \int_z \overleftrightarrow{g}^{h,hq}(z|z')\cdot\vec{h}_r(z'|z_0)\,dz' + \overleftrightarrow{g}^{h,h}(z|z_0)\cdot\hat{\beta}^0 \quad (20)$$

where the depolarizing dyad contribution comes from the direct field term. Substitution of (17) and (18) or (19) and (20) into (4) and (5) yields the desired space-domain Green's dyadics. It is interesting to note that for a source embedded in bianisotropic (or bi-isotropic) media, all four Green's dyadic types have an associated depolarizing contribution, whereas for anisotropic media only the $\{e, e\}$ and $\{h, h\}$ Green's dyadics have such a contribution.

III. Numerical Solution and Results

IE's (14a) and (15a) are solved using a pulse-function, MoM/Galerkin procedure. The unknown fields over the range $0 \le z \le D$ are expanded in a set of pulse functions as

$$\vec{e}_r(z) = \sum_{\beta_e=x,y,z}\sum_{n=1}^{N}\hat{\beta}_e a_n^{\beta_e} p_n(z) e^{-\gamma|z-z_0|}$$
$$\vec{h}_r(z) = \sum_{\beta_h=x,y,z}\sum_{n=1}^{N}\hat{\beta}_h a_n^{\beta_h} p_n(z) e^{-\gamma|z-z_0|} \quad (21)$$

where

$$p_n(z) = \begin{cases} 1 & z_n - \frac{w_n}{2} \le z \le z_n + \frac{w_n}{2} \\ 0 & \text{otherwise} \end{cases}$$

with w_n the width of the nth pulse. The exponential function is incorporated to enhance convergence of (21) for large values of k_x, where $\gamma = p_c$ with p_c defined in the Appendix. Substitution of (21) into (14a) and (15a) and point matching results in a $(6N)\times(6N)$ matrix system $[Z(k_x, k_y)][a] = [b]$ which can be solved for the unknown amplitudes $a_n^{\beta_e}$, $a_n^{\beta_h}$. The spatial integrals associated with expansion can be performed in closed form, so that the MoM matrix entries are determined analytically.

To demonstrate the accuracy and flexibility of the above method, propagation characteristics for several microstrip geometries utilizing complex media are presented and compared to previously published results. Microstrip propagation characteristics were obtained using a space-domain IE, which has as its kernel the bianisotropic medium Green's function of type $\{e, e\}$ previously described. The microstrip IE was solved using a Chebyshev polynomial Galerkin method [22]. Propagation constants k_y were found by an iterative root search. In all of the following results, $N = 20$ uniform-width pulses were used to generate the numerical Green's functions. Because of the smooth nature of the various Green's function components [2], [22], the Green's functions were precomputed at various values of (k_x, k_y) and interpolated.

In Fig. 3 the normalized propagation constant (k_y/k_0) for a microstrip transmission line with an electrically and magnetically anisotropic substrate is shown, with results compared to those of [18]. The constitutive parameters are given as

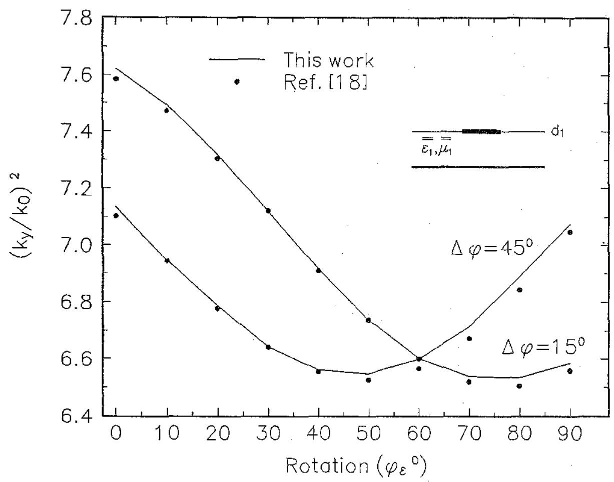

Fig. 3. $(k_y/k_0)^2$ versus anisotropy rotation angle (ϕ_ϵ, $\phi_\mu = \phi_\epsilon + \Delta\phi$) for microstrip on an electrically and magnetically anisotropic substrate at $f = 40$ Ghz, $W = d_1 = 0.1$ cm with $\epsilon_{xx} = 5.12$, $\epsilon_{yy} = 3.4$, $\epsilon_{zz} = 5.12$, $\mu_{xx} = 1.76$, $\mu_{yy} = 1.48$, $\mu_{zz} = 1.62$, and $\theta_\epsilon = \theta_\mu = 0$.

$$[\epsilon] = R^T(\theta_\epsilon, \phi_\epsilon)\begin{bmatrix}\epsilon_{xx} & 0 & 0\\ 0 & \epsilon_{yy} & 0\\ 0 & 0 & \epsilon_{zz}\end{bmatrix}R(\theta_\epsilon, \phi_\epsilon),$$
$$[\mu] = R^T(\theta_\mu, \phi_\mu)\begin{bmatrix}\mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz}\end{bmatrix}R(\theta_\mu, \phi_\mu)$$

where $R(\theta, \phi)$ is an orthogonal rotation matrix which accounts for rotation of the coordinate system wherein the material dyadics take a diagonal form w.r.t. the waveguiding coordinate system [26] and (θ, ϕ) are the usual spherical angles. The matrix R^T is the transpose of R. The rotation of the magnetic anisotropy is different than for the electric anisotropy, with $\theta_\epsilon = \theta_\mu = 0$, $\phi_\mu = \phi_\epsilon + \Delta\phi$. Fig. 4 shows the normalized propagation constant versus frequency for a geometry similar to that of Fig. 3, for two different values of $(\phi_\epsilon, \Delta\phi)$. In both figures good agreement is found with previously published results.

The dispersion curve for the forward and reverse propagation constant of a microstrip transmission line with a ferrite superstrate/isotropic substrate and with a double layer ferrite/isotropic substrate is shown in Fig. 5. The permittivity is isotropic in each region, with anisotropic permeability given by

$$[\mu] = \mu_0 R(\theta, \phi)\begin{bmatrix}\mu & j\kappa & 0\\ -j\kappa & \mu & 0\\ 0 & 0 & 1\end{bmatrix}R^T(\theta, \phi)$$

where $\mu = 1 + (\omega_0\omega_M)/(\omega_0^2 - \omega^2)$, $\kappa = (\omega\omega_M)/(\omega_0^2 - \omega^2)$ with $\omega_0 = \gamma\mu_0 H_0$, $\omega_M = \gamma\mu_0 M_s$ with M_s the material saturation magnetization, H_0 is the dc magnetic bias field, $\gamma = -1.759\times10^{11}$ kg/coul, and (θ, ϕ) are the angles of the applied magnetic field. Results are compared with those of [1]. Good agreement is found except for the reverse propagation constant of the double substrate geometry. This could possible be due to differing number of expansion functions for the microstrip

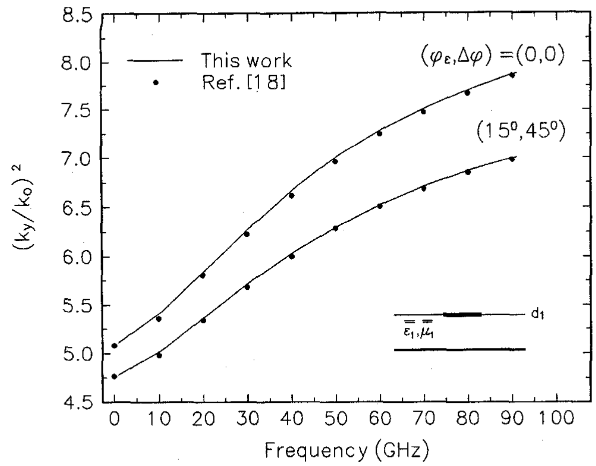

Fig. 4. Dispersion curve for microstrip on an electrically and magnetically anisotropic substrate with $W = d_1 = 0.05$ cm for two values of $(\phi_\epsilon, \Delta\phi)$ with $\epsilon_{xx} = 5.12$, $\epsilon_{yy} = 3.4$, $\epsilon_{zz} = 5.12$, $\mu_{xx} = 1.76$, $\mu_{yy} = 1.48$, $\mu_{zz} = 1.62$, and $\theta_\epsilon = \theta_\mu = 0$.

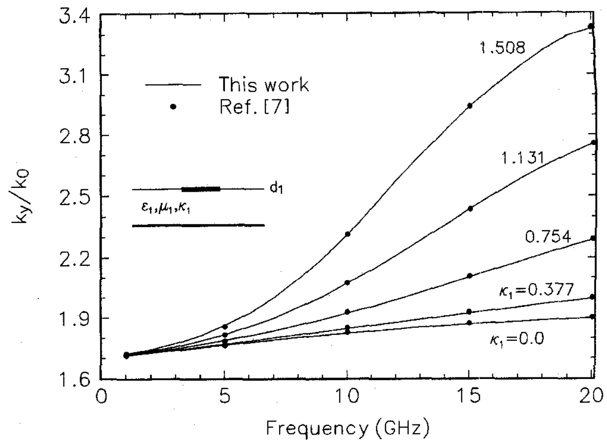

Fig. 6. Dispersion curve for microstrip on a chiral substrate for various values of the chirality parameter. $\epsilon_1 = 4 + \kappa^2$, $\mu_1 = 1$, $d_1 = W = 0.3$ cm, $\xi_1 = -j\sqrt{\epsilon_0\mu_0}\kappa_1$, $\zeta_1 = -\xi_1$. *Note:* Ref. [7] uses a different constitutive model, which corresponds to these numerical values using the model of (2).

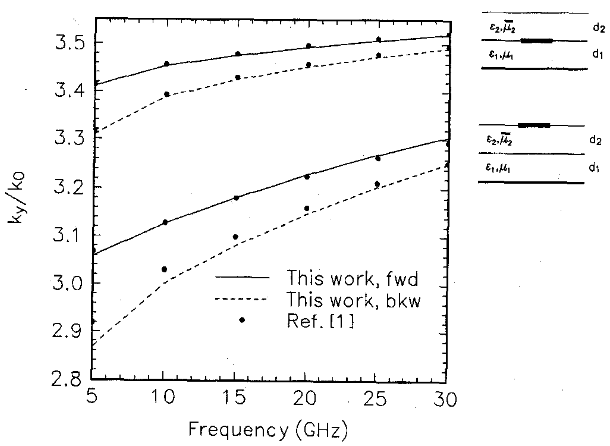

Fig. 5. Dispersion curve for microstrip with a ferrite superstrate/isotropic substrate and with a double layer ferrite/isotropic substrate. $\epsilon_1 = 12.9$, $\epsilon_2 = 12.6$, $\mu_1 = 1$, $\mu_0 M_s = 0.275\,T$, $\omega_0 = 0.1\omega_m$, $d_1 = d_2 = 0.0254$ cm, $W = 0.1016$ cm, $\theta = 90°$, $\phi = 0°$.

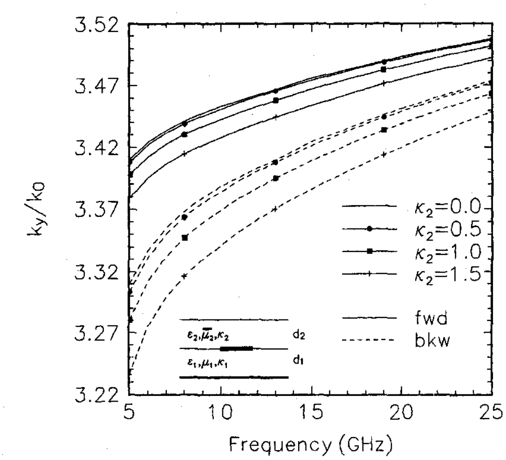

Fig. 7. Dispersion curve for microstrip with a chiroferrite superstrate/isotropic substrate for several different values of chirality parameter κ_2 for $\kappa_1 = 0.0$. $\epsilon_1 = 12.9$, $\epsilon_2 = 12.6$, $\mu_1 = 1$, $\mu_0 M_s = 0.275\,T$, $\omega_0 = 0.1\omega_m$, $d_1 = d_2 = 0.0254$ cm, $W = 0.1016$ cm, $\theta = 90°$, $\phi = 0°$, $\xi_2 = -j\sqrt{\epsilon_0\mu_0}\kappa_2$, $\zeta_2 = -\xi_2$.

current (five are shown here, ten yielded the same results). The authors of [2] also found some values of propagation constant to be lower that those of [1], although they didn't consider the double substrate case.

Fig. 6 shows the dispersion curve for a microstrip line on an isotropic chiral substrate, where $\xi_1 = -j\sqrt{\epsilon_0\mu_0}\kappa_1$, $\zeta_1 = -\xi_1$, for several values of chirality parameter κ_1. Good agreement is seen with the results of [7].

Although not shown here, this method also can be used to obtain the propagation characteristics of bianisotropic slab waveguides. To obtain slab waveguide modes, a root search is performed for values of (k_x, k_y) which force the determinate of the $(6N) \times (6N)$ method of moments matrix to vanish. Results using this method were compared to previously published results for a variety of chiral [27] and achiral planar slabs, where excellent agreement was found.

Finally, some new results for a microstrip transmission line embedded in chiroferrite media are presented in Figs. 7 and 8. The ferrite properties of the superstrate are the same as in Fig. 5, although in Fig. 7 the superstrate also has some nonzero chirality. In Fig. 8 the chirality of the superstrate is fixed, with results shown for several values of the substrate chirality parameter. In both figures it can be seen that the addition of chirality shifts the dispersion curve, although the differential phase shift $(k_y^+ - k_y^-)/k_0$ is not greatly affected by chirality for the range of parameters investigated here.

IV. Conclusion

An integral equation method with numerical solution is presented to determine the complete Green's dyadic for planar bianisotropic media. The Green's function components are determined by the solution of two coupled one-dimensional

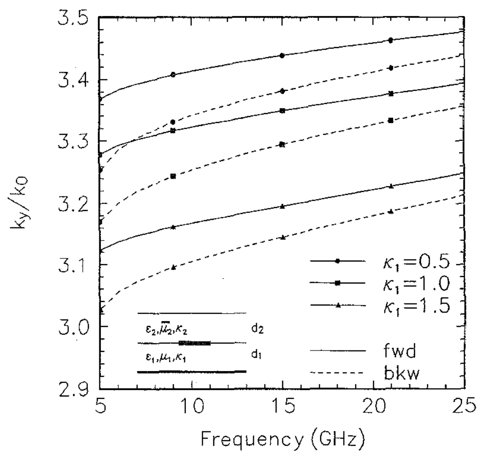

Fig. 8. Dispersion curve for microstrip with a chiroferrite superstrate/bi-isotropic substrate for several different values of chirality parameter κ_1 for $\kappa_2 = 1.0$. $\epsilon_1 = 12.9$, $\epsilon_2 = 12.6$, $\mu_1 = 1$, $\mu_0 M_s = 0.275\,T$, $\omega_0 = 0.1\omega_m$, $d_1 = d_2 = 0.0254$ cm, $W = 0.1016$ cm, $\theta = 90°$, $\phi = 0°$, $\xi_1 = -j\sqrt{\epsilon_0\mu_0}\kappa_1$, $\zeta_1 = -\xi_1$.

IE's, with the regular part determined numerically and the depolarizing dyad contribution determined analytically. This method is appropriate for generating Green's functions for the computation of guided-wave propagation characteristics of conducting transmission lines and dielectric waveguides. The formulation is relatively simple, with the kernels of the IE's to be solved involving only linear combinations of Green's functions for an isotropic half-space. Results for microstrip transmission lines embedded in complex media have been presented to demonstrate the method.

APPENDIX

For a two-region isotropic environment having (ϵ_c, μ_c) for $z > 0$ and (ϵ_s, μ_s) for $z < 0$, the Hertzian potential in the cover region due to an electric source in the cover region in the 2-D spectral plane is given as [28]

$$\vec{\pi}(z) = \int_z \overleftrightarrow{g}_\pi(z|z') \cdot \vec{J}_e(z')\, dz' \tag{A1}$$

where the Green's function $\overleftrightarrow{g}_\pi$ is a Hertzian potential Green's dyadic given by

$$\overleftrightarrow{g}_\pi(z|z') = \overleftrightarrow{I} g^p + (\hat{x}\hat{x} + \hat{y}\hat{y}) g_t^r + \hat{z}[(jk_x\hat{x} + jk_y\hat{y})g_c + g_n^r\hat{z}] \tag{A2}$$

where

$$g^p(z|z') = \frac{\eta_c}{jk_c} \frac{e^{-p_c|z-z'|}}{2(2\pi)p_c} \tag{A3}$$

is the principal (direct) component of potential and

$$\begin{Bmatrix} g_t^r(z|z') \\ g_n^r(z|z') \\ g_c^r(z|z') \end{Bmatrix} = \frac{\eta_c}{jk_c} \begin{Bmatrix} R_t(k_x, k_y) \\ R_n(k_x, k_y) \\ C(k_x, k_y) \end{Bmatrix} \frac{e^{-p_c(z+z')}}{2(2\pi)p_c} \tag{A4}$$

yields the reflected potential. Coefficients are given as

$$R_t = \frac{A_t}{Z^h}, \quad R_n = \frac{A_n}{Z^e}, \quad C = \frac{2(N_{sc}^2 M_{sc}^2 - 1)p_c}{Z^h Z^e} \tag{A5}$$

where

$$A_t = M_{sc}^2 p_c - p_s, \quad A_n = N_{sc}^2 p_c - p_s, \quad Z^e = N_{sc}^2 p_c + p_s, \quad Z^h = M_{sc}^2 p_c + p_s \tag{A6}$$

with $N_{sc}^2 = \epsilon_s/\epsilon_c$, $M_{sc}^2 = \mu_s/\mu_c$, $k_{s,c} = \omega\sqrt{\epsilon_{s,c}\mu_{s,c}}$, $\eta_c = \sqrt{\mu_c/\epsilon_c}$, and $p_{s,c} = \sqrt{k_x^2 + k_y - k_{s,c}^2}$. The resulting fields are

$$\vec{e}(z) = (k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\vec{\pi}(z), \quad \vec{h}(z) = j\omega\epsilon_c\tilde{\nabla} \times \vec{\pi}(z) \tag{A7}$$

where $\tilde{\nabla} = \hat{x}jk_x + \hat{y}jk_y + \hat{z}(\partial/\partial z)$. Properly passing the spatial derivative through the integral in (A1) [24], [25] results in the electric and magnetic dyadic Green's function for an electric source

$$\overleftrightarrow{g}^{e,e}(z|z') = P.V.(k_c^2 + \tilde{\nabla}\tilde{\nabla}\cdot)\overleftrightarrow{g}_\pi(z|z') + \overleftrightarrow{L}\delta(z - z'), \quad \overleftrightarrow{g}^{h,e}(z|z') = \tilde{\nabla} \times \overleftrightarrow{g}_\pi(z|z') \tag{A8}$$

where the depolarizing dyad is $\overleftrightarrow{L} = -\hat{z}\hat{z}/j\omega\epsilon_c$. Invoking duality [26] leads to the other two Green's components as

$$\overleftrightarrow{g}^{h,h}(z|z') = \overleftrightarrow{g}^{e,e}(z|z')|_{\substack{\epsilon\to\mu \\ \mu\to\epsilon}}, \quad \overleftrightarrow{g}^{h,e}(z|z') = -\overleftrightarrow{g}^{e,h}(z|z')|_{\substack{\epsilon\to\mu \\ \mu\to\epsilon}} \tag{A9}$$

where $a \to b$ implies interchanging a and b.

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