On the Nature of Critical Points in Leakage Regimes of a Conductor-Backed Coplanar Strip Line

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***Abstract*— Leaky dominant mode propagation regimes of a conductor-backed coplanar strip line are rigorously analyzed using the concept of critical or equilibrium points from catastrophe and bifurcation theories, in conjunction with a full-wave integral equation solution. The existence of nondegenerate or Morse critical points (MCP's) and degenerate or fold (turning) critical points (FP's) in coupling and leakage regions are associated with the occurrence of improper real and complex (leaky) solutions. The locations and types of critical points determine the stability and instability of the transmission line system with respect to small changes in geometrical parameters. The dispersion behavior of improper real and complex (leaky) solutions are efficiently reproduced in the local neighborhood of MCP's and FP's using a Taylor series expansion about those points. The qualitative and quantitative dynamic behavior of the transmission line modes can be investigated by examining the evolution of nondegenerate and degenerate points versus some structural parameter, such as strip width. The proposed analysis enables the prediction of bifurcation situations and the existence of improper real and complex solutions and gives a complete description of the system's structural behavior.**

I. Introduction

THE LEAKAGE phenomenon for dominant or dominant-like modes on printed transmission lines is a relatively recent discovery. It has been observed that the leakage effects appear at high frequencies for some transmission line structures [1], [2], and at all frequencies for others [3]. The occurrence of leakage leads to a loss of power in transmitting or receiving systems, a decrease in the Q-factor in resonator structures, and cross-talk and coupling between neighboring elements of printed integrated circuits. For example, theoretical and measured results obtained for conductor-backed slot line and coplanar waveguide show serious problems caused by leakage effects: cross-talk, coupling to neighboring lines, and alteration of the wavelength [4]. The above-mentioned detremental effects are usually of the most concern, although beneficial applications of leakage can be found in some novel devices [5] or in the area of leaky-wave antennas.

It has recently been shown that a leaky dominant or dominant-like mode exists on most printed-circuit transmission lines. A leaky dominant mode has been found and experimentally confirmed on microstrip line with an isotropic substrate [6]. It was observed that the leaky dominant mode propagates independently of the bound dominant mode. An independent leaky dominant mode has also been found on stripline with a small air gap [7], where it was determined that an air gap creates conditions for transformation of the TEM mode of traditional stripline into the leaky mode. The analysis of leakage phenomena has also been carried out for stripline with uniaxially anisotropic layers with an air gap [8]. Leakage effects have been observed and investigated for positive and negative uniaxial substrates, where it was found that a proper choice of bonding film material (used to eliminate the air gap) can suppress the undesired leakage.

The leaky dominant mode phenomena has also been investigated for coplanar strip lines [9], [10], coupled slot lines [11], and broadside-coupled microstrip structures [3]. It is shown, for example, that for appropriate geometrical parameters, leakage in broadside-coupled microstrip [3] can exist at all frequencies.

The simultaneous propagation of both the bound and leaky dominant modes in slot and conductor-backed coplanar strip lines has been recently studied [12], [13], and a new improper real (nonphysical) solution has been discovered. It is shown that small changes of geometrical parameters can dramatically change improper real solutions and generate a new improper complex (leaky) solution. The authors contend that the discovered effects can exist in most printed-circuit transmission lines.

The problem of suppressing the leaky modes in printed transmission line structures has been addressed in several papers [14], [15]. It has been proposed that an appropriate combination of geometrical parameters [14] or suitable bonding films or superstrate layers [15] can suppress leakage.

In the present paper, a different view on the dominant or dominant-like leakage mode phenomena is developed. From the standpoint of catastrophe [16], [17] and bifurcation theories [18], [19], the qualitative change in system characteristics by small perturbations can be denoted as a bifurcation or branching. The principles of catastrophe theory have been successfully applied to the analysis of intertype oscillations in open resonators [20] and to the investigation of mode coupling regions in open waveguide resonators [21]. A set of degenerate and nondegenerate points has been determined for complex waves in multilayer cylindrical strip and slot lines [22].

In this paper, we apply the principles of bifurcation and catastrophe theories to the dominant mode leakage phenomena in a printed transmission line structure. The specific example of a conductor-backed coplanar strip line is presented, although the ideas are general and applicable to a wide range of waveguiding structures. Our main goal is to show that certain mode bifurcations and mode coupling behavior can be

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associated with certain types of critical points (CP's). Specifically, the transition from a pair of real (proper-improper or improper–improper) modes to a complex-conjugate improper mode in the "spectral gap" region is associated with the occurrence of a critical fold point (FP). The characteristic intersection of a parabola and a straight line in the spectral gap region is predicted by an analytic quadratic normal form. This behavior in the spectral gap region is universal in the transition from real to complex modes in a variety of waveguiding structures, ranging from planar slabs to more complicated printed conductor and dielectric waveguides.

Another type of CP is associated with characteristic mode coupling behavior, wherein two or more modes approach each other, then separate before crossing. This behavior is found in a variety of shielded and open structures, both for dominant and higher-order modes [3], [23], [24], and can also be predicted by a quadratic normal form. Since the occurrence of CP's can be associated with interesting modal features such as intermode coupling and leakage, the determination of the location and types of CP's in a region of interest may facilitate the rapid location of interesting modal behavior. The dispersion behavior in the vicinity of a CP can be efficiently reconstructed using a Taylor series expansion about that point, enabling the efficient determination of modal behavior before evaluating a full-wave solution.

The reader should be aware that different terminology is used in catastrophe and bifurcation theories. For convenience, we will use the label "critical point" (CP) to describe certain points in (κ, f) space which are associated with coupling and modal bifurcations.

II. Theory

In order to investigate the association of CP's with modal behavior, the example of a conductor-backed coplanar strip line shown in the insert of Fig. 1(a) is considered. Full-wave results are generated using an electric-field integral equation technique similar to [25]. Enforcement of the boundary condition requiring that the tangential electric field vanish on the surface of all conducting strips results in a coupled system of homogeneous integral equations in the axial-Fourier transform domain $(z \leftrightarrow k_z)$

$$\hat{\alpha}_m \cdot \sum_{n}^{N} \int_{x_n} \overline{g}(x|x'; k_z) \cdot \vec{J}_n(x')\, dx' = 0, \quad x \in x_m, \quad m = 1, \cdots, N \tag{1}$$

where $\hat{\alpha}_m$ is a unit vector tangent to the mth strip, $\overline{g}(x|x'; k_z)$ is the electric Green's dyadic, $\vec{J}_n(x')$ is the unknown transform-domain surface current on the nth strip, and k_z is the unknown propagation constant. The Green's dyadic is given as a Sommerfeld-type integral over the transverse transform variable k_x. The unknown longitudinal and transverse surface currents are expanded as a series of Chebyshev polynomials, and a Galerkin solution converts the system of integral equations into a matrix system $A(k_z, f)X = 0$, where f represents frequency and X represents the vector of unknown coefficients of the current density. A root search is performed to determine the value of propagation constant that forces $\det[A] = 0$. For proper bound modes the path of integration for the Green's function inversion is along the real k_x-axis, whereas for improper modes the path is deformed into the complex k_x-plane [25].

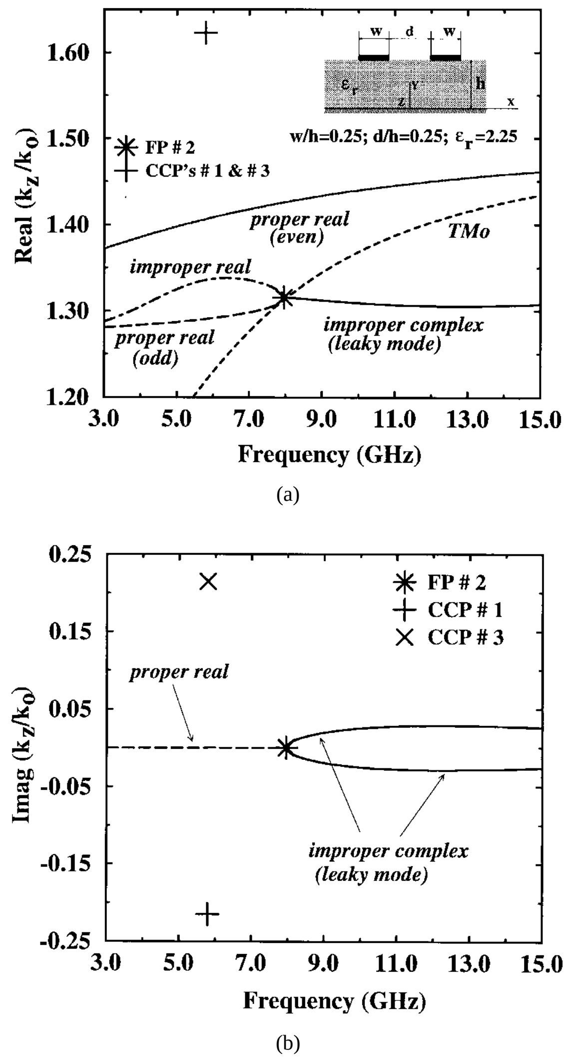


Fig. 1. Dispersion characteristics for the (a) normalized phase Real(k_z/k_0) and (b) leakage Imag(k_z/k_0) constants in the case of $w/h = 0.25$ for a conductor-backed coplanar strip line. An FP (FP ♯2), which is associated with the fold catastrophe, is found in the spectral gap region.

Equation (1) can be used to generate full-wave results for the considered transmission line, and similar formulations can be developed to model other printed geometries. This system of equations also forms the basis for the determination of various CP's associated with the waveguiding structure. To determine CP's, consider the smooth function $H(\kappa, f) = \det[A(\kappa, f)]$ in the complex domain of spectral parameters, where κ is the normalized propagation constant k_z/k_0. Assume that $H(\kappa, f)$ is an analytic function in the functional space C^2 of two complex variables κ and f. Investigating behavior of $H(\kappa, f)$ in a certain domain $D \subset C^2$ and solving the problem for a discrete set of solutions of the characteristic equation $H(\kappa, f) = 0$, we determine a set of regular points of $H(\kappa, f)$, which are the modal solutions of the transmission line. One set of CP's of $H(\kappa, f)$ can be obtained when a necessary condition of the function's extremum is satisfied. Define a set of CP's [17] $\Omega_{\rm CP}$ of a mapping H: $C^2 \rightarrow C$ by

$$\nabla_{\kappa,f} H(\kappa, f) = 0. \tag{2}$$

Equation (2) is equivalent to the set of nonlinear equations of partial derivatives of $H(\kappa, f)$ with respect to κ and f: $H'_\kappa(\kappa, f) = 0$ and $H'_f(\kappa, f) = 0$. These CP's usually determine local maxima, local minima, and saddles. If $\nabla_{\kappa,f} H(\kappa, f) \neq 0$ then the functional behavior in the vicinity of a regular point can be easily obtained using the implicit function theorem [16], where a unique curve $\kappa = \kappa(f)$ or $f = f(\kappa)$ through a regular point can be reconstructed. If $\nabla_{\kappa,f} H(\kappa, f) = 0$ then the implicit function theorem is no longer applicable. However, if the Hessian matrix is nonsingular with the Hessian determinant $\det[H''_{\imath,\jmath}(\kappa, f)]$ $(\imath, \jmath = \kappa, f)$

$$\xi = [H''_{\kappa\kappa} H''_{ff} - H''_{\kappa f} H''_{f\kappa}]|_{(\kappa_m, f_m)} \neq 0 \tag{3}$$

then, according to the Morse L emma [16], there is a smooth change of coordinates such that the function H can be locally represented by a quadratic canonical form. In other words, an analytic representation of $H(\kappa, f)$ exists in the local neighborhood of CP's defined by the set of equations (2) and (3). A set of these points, called nondegenerate or Morse CP's (MCP's), is denoted $\Omega_{\rm MCP}$: $\{(\kappa_m, f_m)\}$, where $\Omega_{\rm MCP} \subset \Omega_{\rm CP}$. The concept of MCP's is related to structural stability of a system in a local region. It is shown [26] that $H(\kappa, f)$ is structurally stable at the MCP (κ_m, f_m). A function, whose set of CP's are nondegenerate (MCP's), is called a Morse function and its structural stability is guaranteed.

If the Hessian determinant ξ is positive, nondegenerate points define local minimum or maximum [16], [26]. The case when ξ is negative is related to the universal mode coupling behavior observed in many waveguiding structures [3], [23], [24]. Local dynamical behavior of the function $H(\kappa, f)$ in a bifurcation region can be determined by the simplest equations called normal forms [19]. It can be shown [18] that in the local neighborhood of the MCP (κ_m, f_m) the normal form is represented as the intersection of a saddle surface $H(\kappa, f)$ and the mode coupling factor, $H(\kappa_m, f_m)$:

$$(\kappa - \kappa_m)^2 - (f - f_m)^2 = H(\kappa_m, f_m). \tag{4}$$

Equation (4) determines a set of hyperbolas centered at (κ_m, f_m) in the coordinate system (κ, f). These hyperbolas form the characteristic behavior typically seen when modes are coupled together and, in this example, appear in the mode interaction of the two real improper modes as will be shown later, e.g., Fig. 2(a). If the coupling factor $H(\kappa_m, f_m)$ is equal to zero, then the solution is locally reproduced as two straight lines defined by equations $\kappa = \kappa_m + (f - f_m)$ and $\kappa = \kappa_m - (f - f_m)$. The point of intersection of these lines is a double point of $H(\kappa, f)$, which locally defines a double-point bifurcation [18]. The function $H(\kappa, f)$ is unstable at this point, and the system's characteristics are qualitatively changed for small perturbations. If $H(\kappa_m, f_m) \neq 0$ the double-point bifurcation is broken into isolated solutions qualitatively determined by the normal form (4).

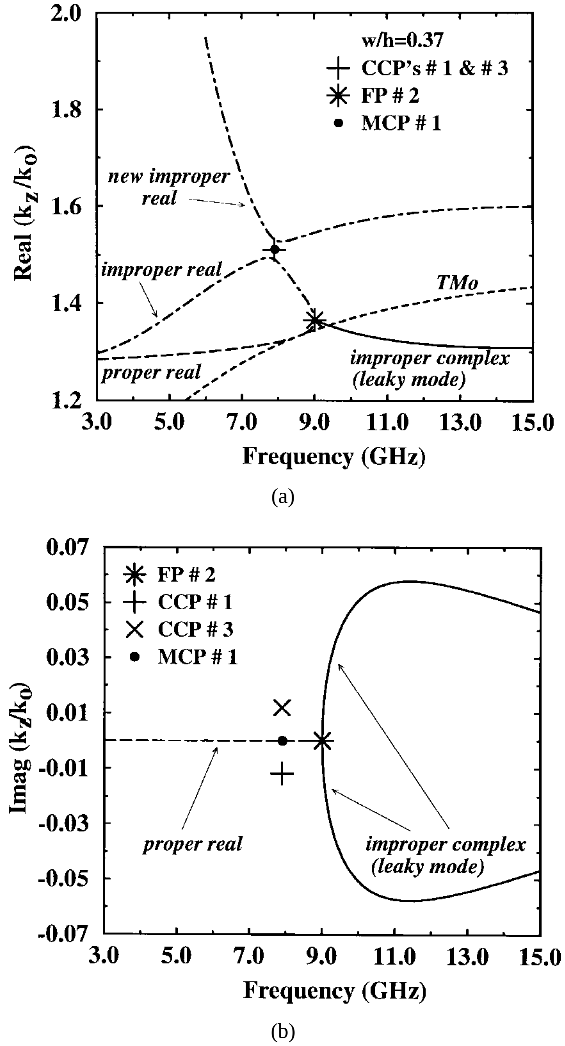


Fig. 2. Full-wave results for (a) normalized phase and (b) leakage constants versus frequency for $w/h = 0.37$. A pair of complex conjugate CP's (CCP's $\sharp 1$ and $\sharp 3$) and a Morse Critical Point (MCP $\sharp 1$) define the mode-coupling region of improper real solutions.

If the MCP is obtained in the vicinity of the mode coupling region, the qualitative and quantitative local structure can be reproduced using a Taylor series expansion:

$$\begin{aligned} H(\kappa, f) = & H(\kappa_m, f_m) + H'_\kappa(\kappa - \kappa_m) + H'_f(f - f_m) \\ & + \tfrac{1}{2} H''_{\kappa\kappa}(\kappa - \kappa_m)^2 + H''_{\kappa f}(\kappa - \kappa_m)(f - f_m) \\ & + \tfrac{1}{2} H''_{ff}(f - f_m)^2 \\ & + o[(|\kappa - \kappa_m| + |f - f_m|)^2] = 0 \end{aligned} \tag{5}$$

where $o[(|\kappa - \kappa_m| + |f - f_m|)^2]$ are cubically small terms and all partial derivatives are calculated at (κ_m, f_m). According to the condition (2), the partial derivatives $H'_{\kappa|_{(\kappa_m, f_m)}}$ and $H'_{f|_{(\kappa_m, f_m)}}$ are equal to zero and the local structure is completely defined by coefficients of the Hessian matrix and the coupling factor $H(\kappa_m, f_m)$, which determines the intensity of the mode coupling.

A different type of critical point, called a fold or turning point [19], is related to the leakage phenomena observed in many waveguiding structures. A set of fold or turning points (FP's) $\Omega_{\rm FP}$: $\{(\kappa_f, f_f)\}$, obeys the following set of equations

[19]:

$$H(\kappa,f)|_{(\kappa_f,f_f)} = H'_\kappa(\kappa,f)|_{(\kappa_f,f_f)} = 0$$
$$\delta = H''_{\kappa\kappa}(\kappa,f)H'_f(\kappa,f)|_{(\kappa_f,f_f)} \neq 0. \quad (6)$$

It can be shown [19] that if (κ_f, f_f) is a fold or turning point, then, locally, $H(\kappa,f)$ is equivalently represented in the normal form

$$(\kappa-\kappa_f)^2 + (f-f_f), \quad \text{for} \quad \delta>0,$$
$$(\kappa-\kappa_f)^2 - (f-f_f), \quad \text{for} \quad \delta<0. \quad (7)$$

for the $\delta>0$ case with $f<f_f$, two branching solutions $\mathrm{Re}[\kappa(f)]$ of $(\kappa-\kappa_f)^2+(f-f_f)=0$ generate a parabola, and for $f>f_f$ two equal solutions $\mathrm{Re}[\kappa(f)]$ exist as a straight line $\kappa(f)=\kappa_f$. This predicts the characteristic intersection of a parabola and a straight line that occurs at a point of mode bifurcation, such as in the spectral gap region. When $f=f_f$ there is only one solution (κ_f,f_f) (FP), which occurs at the intersection point of the two curves. Obviously $\mathrm{Im}[\kappa(f)]$ for $f<f_f$ yields the solution $\kappa(f)=0$, and for $f>f_f$ two branching solutions form a parabola in the imaginary plane of $\kappa(f)$. The above description clearly applies to the situation shown in Fig. 1(a) and (b), which will be discussed later. The normal form $(\kappa-\kappa_f)^2-(f-f_f)$ can be investigated using a similar evaluation. It is important to emphasize that the normal form (7) is qualitatively related to the modal behavior of improper real and complex solutions in the vicinity of an FP. The FP satisfies (1), whereas the normal form (7) indicates that it will occur at a point of modal branching. A Taylor series expansion (5) reproduces quantitatively the local structure of the function $H(\kappa,f)$ in the vicinity of an FP (κ_f,f_f). The fold catastrophe is locally determined by $H'_f(\kappa,f)$ and coefficients of the Hessian matrix at (κ_f,f_f). It should be noted that the system of (6) defines both real and complex CP's. It was found that for this example the CP's were real or occurred in complex-conjugate pairs. The real valued and complex-conjugate points will henceforth be denoted as FP's and CCP's, respectively. The normal form (7) is applicable for real valued FP's, where δ will be real.

III. Numerical Results

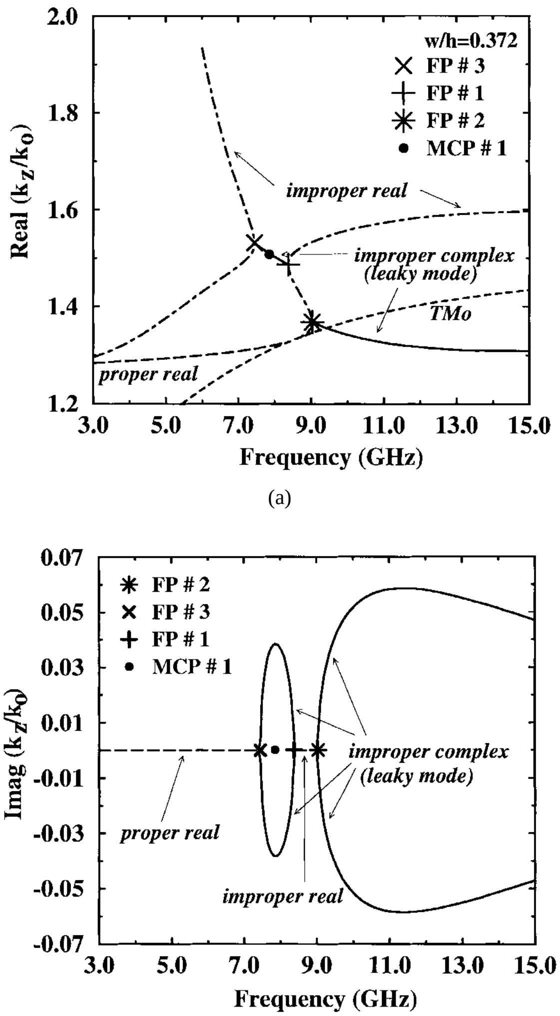
The spectrum of all possible solutions for the conductor-backed coplanar stripline geometry presented in [12] and [13], including real improper and proper modes and improper complex (leaky) modes, has been generated using the above-mentioned rigorous electric field integral equation formulation. In the results to follow, the substrate thickness is set as $h=1$ cm. For the frequencies considered, this corresponds to the geometry investigated in [12] and [13]. In all of the following figures, modal solutions resulting from the set of integral equations (1) are presented, except where results are explicitly denoted in the figure legend as "local structure." In this case quantitative dispersion characteristics have been generated by applying the quadratic formula to Taylor series (5), resulting in an analytic formula for $\kappa(f)$ in terms of the various partial derivatives of $H(\kappa,f)$. The derivatives in the expansion (5) have been calculated numerically using finite difference approximations. Since rigorous results for the considered structure have appeared in [12] and [13], the results presented here are intended to connect the previously described behavior with the presence of certain types of CP's defined by (6) or (2) and (3).

The occurrence of these CP's is intimately connected with the observed physical phenomena, and the qualitative behavior is shown to obey the simple quadratic normal forms (4) or (7). These normal forms help to provide insight into the observed interesting modal behavior in the mode coupling regions, and in the generation of leakage.

Fig. 1 demonstrates dispersion characteristics for the normalized phase constant Real(k_z/k_0) and normalized leakage constant Imag(k_z/k_0) for narrow strips with $w/h=0.25$. For completeness, the even (proper) mode is shown along with the various proper and improper odd modes. Attention will be focused on the odd modes for the remainder of the paper. The behavior of the function $H(\kappa,f)$ has been investigated in the vicinity of the spectral gap region using the concept of CP's defined by (6). A fold real valued point (FP $\sharp 2$) has been found in the spectral gap region at the intersection of improper real and improper complex solutions, having coordinates $(\kappa_{f2},f_{f2})=(1.3156,7.9536)$, where frequency is given in gigahertz.

The qualitative local structure of $H(\kappa,f)$ in the spectral gap region is determined by the normal form (7) with $\delta>0$, as previously described. A Taylor series expansion (5) gives quantitative behavior of $H(\kappa,f)$ in the vicinity of the FP $\sharp 2$, generating improper and proper real solutions and the improper complex (leaky mode) solution with good accuracy, although the result is not included here. A pair of CCP's $\sharp 1$ and $\sharp 3$), obtained as the solution of (6), is presented in Fig. 1, having coordinates $(\kappa_{c1,3},f_{c1,3})=(1.6229\mp \jmath 0.2149, 5.7915\pm \jmath 3.8442)$. These CCP's are not associated with leakage in this figure, but will become a pair of real FP's as strip width increases, resulting in leakage.

Figs. 2 and 3 show that a small change in strip width from $w/h=0.370$ to $w/h=0.372$ results in an enormous change in the improper real solutions and generates a new nonphysical improper complex (leaky) solution. This qualitative change in structural behavior is explained by the transformation of the complex conjugate points into two real valued FP's. It is found that the pair of CCP's $\sharp 1$ and $\sharp 3$ with coordinates $(\kappa_{c1,3},f_{c1,3})=(1.5107\mp \jmath 0.0119, 7.8937\pm \jmath 0.2461)$, defined by the formulation (6), exists in the mode-coupling region of improper and new improper real solutions as depicted in Fig. 2. The local structure can be efficiently reproduced using a Taylor series expansion (5) in the neighborhood of CCP's $\sharp 1$ and $\sharp 3$. Small changes of w/h can change the type of the CP's. It is observed that CCP's $\sharp 1$ and $\sharp 3$ transition into FP's $\sharp 1$ and $\sharp 3$ with coordinates $(\kappa_{f1},f_{f1})=(1.4865,8.3688)$ and $(\kappa_{f3},f_{f3})=(1.5315,7.4473)$ upon a slight increase in strip width, as shown in Fig. 3. The transformation of CCP's into real valued FP's is found to define new structural behavior, and is associated with the creation of an improper complex mode. The process of the transformation of CP's is related to structural instability. In other words, small changes in a geometrical parameter lead to new qualitative behavior of the



(a)

(b)

Fig. 3. Results for the case when strip width is increased slightly to $(w/h = 0.372)$. The occurrence of a new leaky mode is associated with the transition of CCP's into real valued FP's.

system. As a result, the occurrence of a new improper complex (leaky mode) solution and dramatic qualitative changes of the improper real solutions, is connected with the transition of a conjugate pair of complex CP's into real valued FP's and the formation of a new local structure. The real valued FP $\sharp 2$ obtained in the spectral gap region is relatively unaffected by the increase in strip width, with coordinates $(\kappa_{f2}, f_{f2}) = (1.3654, 9.0028)$ in the case $w/h = 0.370$ and $(\kappa_{f2}, f_{f2}) = (1.3676, 9.0347)$ in the case $w/h = 0.372$.

The phenomena wherein slight changes in strip width result in significant changes in modal behavior, including the generation of new improper modes, were discussed in [12] and [13]. This can be explained by consideration of the CCP $\rightarrow$ FP transition described above, in conjunction with a consideration of other CP's that occur in the mode-coupling region. In addition to the CP's satisfying (6), it was discovered that a real MCP $\sharp 1$, defined by the set of (2) and (3), exists in the interaction region of improper real solutions with coordinates $(\kappa_{m1}, f_{m1}) = (1.5110, 7.9101)$, as demonstrated in Fig. 2. A small increase in strip width slightly changes the location

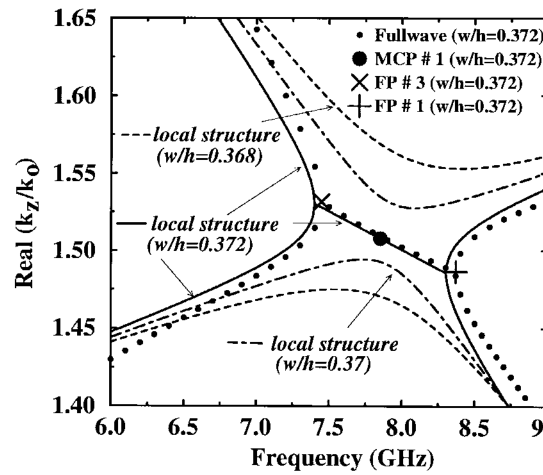


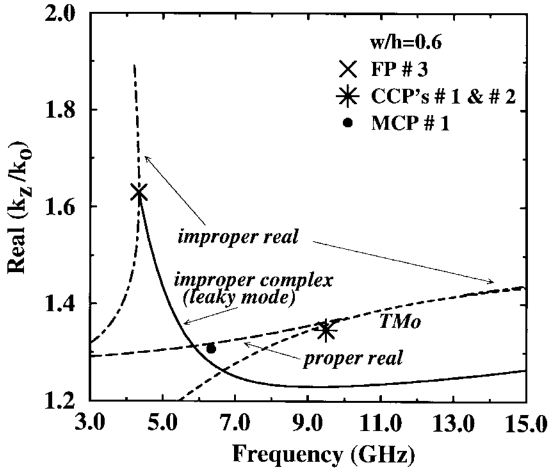
Fig. 4. A family of local structures defined by MCP's for different w/h, which is the result of intersection of a saddle surface $H(\kappa, f)$ and the planes $H(\kappa_m, f_m) = \text{constant}$.

of the MCP $\sharp 1$ to $(\kappa_{m1}, f_{m1}) = (1.5079, 7.8516)$ (Fig. 3) in comparison with significant changes of CCP's $\sharp 1$ and $\sharp 3$.

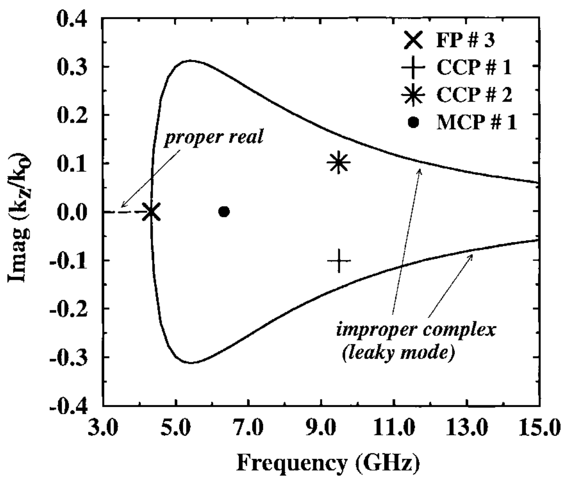
According to [26], nondegenerate points (MCP's) define a local minimum, maximum, and different kinds of saddles. If the Hessian determinant is strictly negative, then a MCP is a saddle point. Fig. 4 demonstrates a family of local structures generated about their MCP's versus strip width w/h for values considered in Figs. 2 and 3. For our purposes, the term "local structure" will refer to the qualitative behavior predicted by the appropriate normal form (4) or (7), which is quantified numerically by the Taylor series (5).

According to definitions given in Section II, we can conclude that the local structures shown in Fig. 4 are the intersection of a saddle surface $H(\kappa, f)$ and the planes $H(\kappa_m, f_m) =$ constant. All presented local structures are determined by expanding about saddle points (MCP's $\sharp 1$). Local structures associated with the transition region from a pair of real improper modes to a complex-conjugate improper mode could have been generated about the FP's as well. It was found that between $w/h = 0.370\,40$ and $w/h = 0.370\,45$ there is a double point that determines structural instability. The local structure at a double point (d.p.) is the intersection of straight lines with $H(\kappa, f)|_{d.p.} = 0$. The coordinates of CP's $\sharp 1$ and $\sharp 3$, and MCP $\sharp 1$ are equal here. Before and after this point a system is structurally stable. The enormous change in the local structure when strip width is increased from $w/h = 0.370$ to $w/h = 0.372$ is explained by the sign of $H(\kappa, f)$ at (κ_m, f_m). Before a double point $H(\kappa_m, f_m)$ is positive, and after $H(\kappa_m, f_m)$ is negative.

According to the normal form (4), the intersection of a saddle surface $H(\kappa, f)$ with a positive or negative $H(\kappa_m, f_m)$ determines a local structure, like the one shown in Fig. 4 for $w/h = 0.370$ or $w/h = 0.372$, respectively. In effect, a change in the sign of $H(\kappa_m, f_m)$ determines which opposing quadrants formed by the double-point bifurcation (locally intersected straight lines at a double point with $H(\kappa_m, f_m) = 0$) the modal solution will occupy, as explained in the discussion succeeding (4). It should be noted that the improper real solutions demonstrated in Fig. 4 in the local vicinity of a



(a)



(b)

Fig. 5. Dispersion results for wider strips $(w/h = 0.6)$, where the simultaneous propagation of bound and leaky modes is observed. A set of real FP's and CCP's is obtained.

double point can be adequately reproduced by the normal forms (4) and (7).

Fig. 5 shows that for wider strips $(w/h = 0.6)$ the spectral gap disappears and the simultaneous propagation of bound and leaky modes occurs [12]. A set of CP's has been obtained for this geometry. A pair of CCP's $\sharp 1$ and $\sharp 2$ with coordinates $(\kappa_{c1,2}, f_{c1,2}) = (1.3459 \mp \jmath 0.1012, 9.4987 \mp \jmath 3.8399)$ is determined as the solution of (6). The real FP $\sharp 3$ is obtained in the intersection point of improper real and complex solutions, and has coordinates $(\kappa_{f3}, f_{f3}) = (1.6303, 4.3357)$. The real MCP $\sharp 1$ has been moved to the position $(\kappa_{m1}, f_{m1}) = (1.3068, 6.3346)$. It should be noted that the normal form (4) represents the qualitative dynamical behavior of dispersion curves in a local neighborhood of MCP's. This corresponds to small perturbations which result in small coupling effects. The qualitative and quantitative local structure can be reproduced using a Taylor series expansion (5) in the vicinity of MCP's for a small value of the coupling factor $H(\kappa_m, f_m)$.

The qualitative and quantitative dynamic behavior of the coplanar line is investigated by examining the evolution of MCP's, CCP's, and FP's versus strip width. Fig. 6 shows the evolution of Real(k_z/k_0) and Real(f) of CP's versus strip width from $w/h = 0.2$ to $w/h = 0.6$. Marked arrows are related to the direction of the critical point movement. The pair of CCP's $\sharp 1$ and $\sharp 3$ exists from $w/h = 0.2$ up to a double point A, which is between $w/h = 0.37040$ and $w/h = 0.37045$, as discussed previously. For instance, Fig. 2 depicts the modal structure at a value of w/h just prior to point A, whereas Fig. 3 depicts the structure at a w/h value slightly greater than that at point A. After point A two real valued FP's $\sharp 1$ and $\sharp 3$ appear and move in opposite directions.

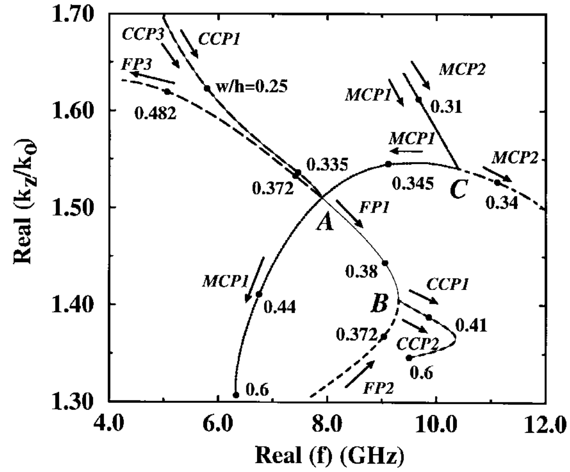


Fig. 6. The evolution of Real(k_z/k_0) and Real(f) of CP's and MCP's within the range 0.2 and 0.6 of w/h. Singular points A, B, and C determine the points of rapid changes in system characteristics, at which points the transmission line structure is unstable.

It should be noted that an improper complex (leaky mode) solution occurs at point A. The transformation of real valued FP's $\sharp 1$ and $\sharp 2$ into CCP's appears at a singular point B, which is found within the range 0.3855 and 0.3856 of w/h values. The improper real solution that exists between the two leaky mode regimes is maintained between points A and B and disappears at a singular point B. Fig. 3 demonstrates the modal behavior before point B (just after point A), whereas Fig. 5 shows the modal structure after point B. The improper complex (nonphysical) leaky mode solution, generated at point A, joins the improper physical leaky mode defined by the FP $\sharp 2$ at point B, where we use the term physical and nonphysical as discussed by Oliner [12], [13].

According to Fig. 6, MCP $\sharp 1$ passes through a double point A, which determines a point of structural instability. A third singular point C has been found in the range of 0.3385 and 0.3390 values of strip width. A pair of complex conjugate MCP's $\sharp 1$ and $\sharp 2$ transitions into real valued MCP's at this point, and the local structure is qualitatively changed. The evolution of Imag(k_z/k_0) and Real(f) of CP's and MCP's versus strip width is shown in Fig. 7. The presented imaginary characteristics are related to the real ones demonstrated in Fig. 6. It can be observed that two CCP's $\sharp 1$ and $\sharp 3$ are transformed into real FP's moving apart from point A. The improper real solution (no imaginary part) exists between points A and B. A conjugate pair of complex CP's $\sharp 1$ and $\sharp 2$ is generated at point B. Complex MCP's $\sharp 1$ and $\sharp 2$ are joined at point C and move in opposite directions.

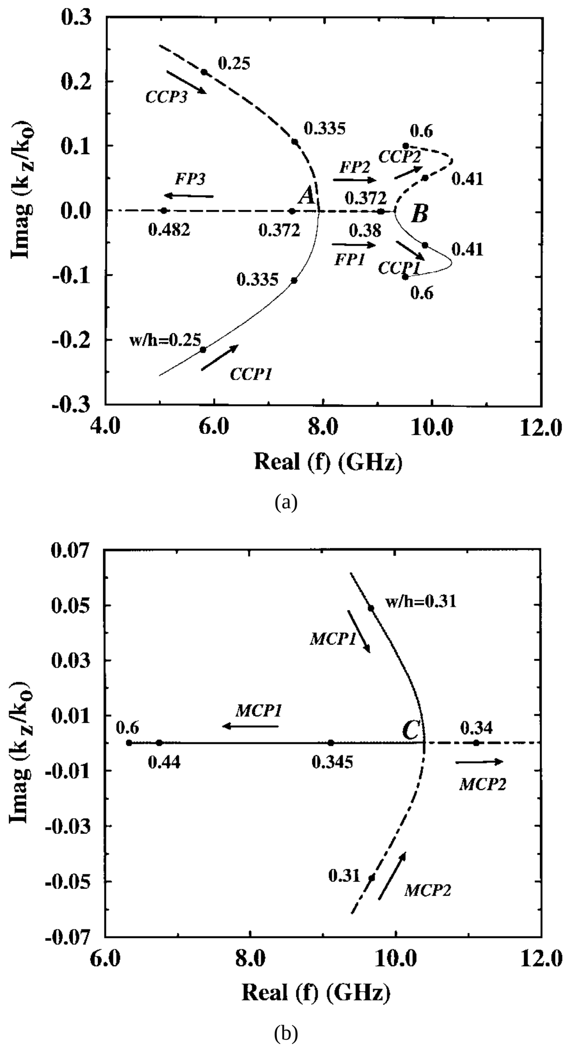


Fig. 7. The evolution of Imag(k_z/k_0) and Real(f) of (a) CP's and (b) MCP's within the range of w/h related to Fig. 6. The improper real solution exists between singular points A and B.

Figs. 6 and 7 carry important information about regions of structural instability (Points A, B, and C), and regions of the appearance and disappearance of improper real and complex solutions.

IV. Conclusion

The concept of CP's from catastrophe and bifurcation theories is a powerful approach for examining qualitative and quantitative dynamic behavior of a system. Based on the determination of degenerated or nondegenerated CP's, the evolution of CP's can be generated and regions of existence, appearance, and disappearance of improper real and complex (leaky mode) solutions can be predicted. It is found that a degenerate point, determining the fold catastrophe, is associated with the universal spectral gap behavior found in the transition from a bound to a leaky regime in various types of waveguiding structures. The proposed analysis enables determination of regions of structural stability and instability. It is shown that changes in the types of CP's are related to qualitative changes of structural characteristics of a system. The presented method necessitates the use of a full-wave solution for determination of the CP's, but allows for the rapid location of regions in reciprocal (κ, f) space which contain interesting and important features associated with leakage.

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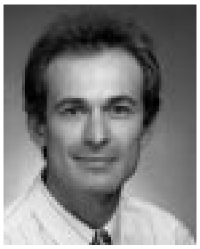

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