Full-Wave Perturbation Theory for the Analysis of Coupled Microstrip Resonant Structures

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Abstract—**A full-wave perturbation theory for the system of N coupled microstrip disk structures is presented. The theory is based on the electric field integral equation description of the circuit, which includes all of the wave phenomena associated with the conductors and the surrounding media. This method is suitable for quantification of nearly degenerate coupling between open microstrip disks, yielding the complex system eigenmodes. For the case of two coupled disks, the perturbation theory analytically separates, though simultaneously solves for, the symmetric and antisymmetric system eigenmodes. The development of the perturbation theory leads to good physical insight for this mode splitting phenomena. Numerical results obtained with the perturbation theory agree well with those obtained by a more accurate method of moments solution to the coupled set of electric field integral equations, as well as with experimental data.**

I. Introduction

PRINTED microstrip disk structures are used extensively as microwave and millimeter-wave integrated-circuit components. A number of theoretical methods have been developed to analyze the resonant characteristics of these structures, the most accurate techniques arising from rigorous full-wave spectral or space domain methods which account for all wave phenomena associated with the structures and the surrounding media [1]–[7]. Application of these methods for the study of system resonances of coupled microstrip elements is quite involved both analytically and numerically, and becomes prohibitive as the number of elements increases.

In this paper, a full-wave perturbation theory for the analysis of the eigenmodes of the system of N coupled microstrip disk structures in an open (unshielded) environment is developed, based upon a plane-wave spectral domain formulation of coupled electric field integral equations (EFIE's). The coupled set of EFIE's are solved in an efficient manner by approximating the system eigenmode currents of loosely coupled disks by the eigenmode currents of the isolated disks, forming a perturbation matrix eigenvalue equation. The method remains tractable as the number of elements increases, and provides a simple formulation for the study of nearly degenerate, multi-disk coupling. The requirement of nearly degenerate coupling is not overly restrictive, as many systems of coupled printed circuit antennas and resonators fall into this category.

Numerical results obtained with the perturbation theory are compared with the method of moments (MoM) solution to the coupled set of EFIE's. An experimental implementation of two coupled disks is investigated with a network analyzer. Resonant system eigenmodes are measured and found to be in good agreement with the perturbation theory and the MoM solution.

Manuscript received October 15, 1991; revised January 21, 1992.

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IEEE Log Number 9201715.

II. Problem Formulation

The configuration of coupled microstrip disks is shown in Fig. 1 for $N = 2$ narrow rectangular structures, although this procedure may be applied to other printed disk shapes. The conducting disks are located in the cover region of the tri-layered conductor/film/cover environment, at the film/cover interface. The coordinate origin is chosen at the film/cover interface, with y normal to and x, z tangential to that interface. The cover and film regions are characterized by $\epsilon_i = n_i^2\epsilon_0$ and $\mu_i = \mu_0$ for $i = f, c$, where n_i is the refractive index of the ith layer. The wavenumber and intrinsic impedance of each layer are $k_i = n_i k_0$ and $\eta_i = \eta_0/n_i$ where (k_0, η_0) are their free-space counterparts. In this work, the cover region is assumed to be free space.

The electric field in the cover is obtained from the Hertzian potential there [8], and implementation of the boundary condition for tangential electric fields at the conducting disk's surfaces leads to the coupled set of EFIE's for surface current $\vec{K}_n$ induced on the nth disk by impressed excitation $\vec{E}^i$ as

$$\hat{t}_m \cdot \sum_{n=1}^{N} \int_{S_n} \vec{G}^e(\vec{r} \mid \vec{r}\,') \cdot \vec{K}_n(\vec{r}\,')\, dS' = \frac{-jk_c}{\eta_c}\hat{t}_m \cdot \vec{E}^i(\vec{r}) \qquad \forall \vec{r} \in S_m$$
$$m = 1, 2, \cdots, N \tag{1}$$

where $\hat{t}_m$ is a unit vector tangent to the mth disk and $\vec{r} = \hat{x}x + \hat{y}y + \hat{z}z$ is the 3-D position vector. The electric dyadic Green's function is given in [8], and will not be repeated here. The coupled set of EFIE's (1) provide the fundamental resource for the investigation of EM phenomena in multi-disk systems.

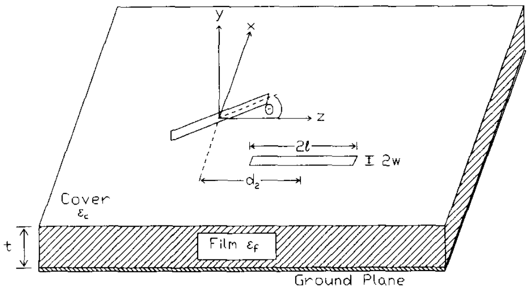

Fig. 1. Configuration of two identical coupled microstrip disks.

Eigenmode currents are associated with pole singularities at complex system resonant frequencies $\omega = \omega_q$ in the temporal frequency domain. For ω near a pole frequency ω_q, the current on the nth element of the N disk system can be approximated by the singularity expansion [9]

$$\vec{K}_n(\vec{r}, \omega) \approx \sum_q \frac{a_q \vec{k}_{nq}(\vec{r})}{(\omega - \omega_q)} \tag{2}$$

where $\vec{k}_{nq}(\vec{r})$ is the eigenmode current of the qth system mode on the nth disk. Exploiting expansion (2) in the coupled EFIE's (1) and noting that $\vec{E}^i$ is regular as $\omega \to \omega_q$ results in the homogeneous EFIE system

$$\hat{t}_m \cdot \sum_{n=1}^{N} \int_{S_n} \overleftrightarrow{G}^e(\vec{r} \mid \vec{r}\,'; \omega) \cdot \vec{k}_{nq}(\vec{r}\,')\, dS' = 0 \qquad \forall \vec{r} \in S_m$$
$$m = 1, 2, \cdots, N \tag{3}$$

with non-trivial solutions for $\omega = \omega_q$, which defines the qth system mode with natural frequency ω_q and element current distribution $\vec{k}_{nq}$.

The direct solution of (3) by the MoM is examined in Section IV, although it is found that this method is numerically inefficient due to the difficulty of evaluating the Sommerfeld-type integrals associated with $\overleftrightarrow{G}^e$. A coupled-mode perturbation approximation, based upon the isolated natural-mode currents of coupled-system elements is consequently prompted.

III. Coupled-Mode Perturbation Theory

In this section the coupled-mode perturbation theory is developed from the set of coupled EFIE's (3). The testing operator $\int_{S_m} dS\, \vec{k}^{(0)}_{mq}(\vec{r}) \cdot$ is applied to EFIE's (3), where $\vec{k}^{(0)}_{mq}$ is the resonant current of the qth mode on the mth isolated disk, satisfying the isolated EFIE

$$\hat{t}_m \cdot \int_{S_m} \overleftrightarrow{G}^e(\vec{r} \mid \vec{r}\,'; \omega_q^{(0)}) \cdot \vec{k}^{(0)}_{mq}(\vec{r}\,')\, dS' = 0 \qquad \forall \vec{r} \in S_m. \tag{4}$$

Exploiting reciprocity of $\overleftrightarrow{G}^e$, $G^e_{\alpha\beta}(\vec{r} \mid \vec{r}\,') = G^e_{\beta\alpha}(\vec{r}\,' \mid \vec{r})$, leads to

$$\sum_{n=1}^{N} \int_{S_n} dS'\, \vec{k}_{nq}(\vec{r}\,') \cdot \int_{S_m} \overleftrightarrow{G}^e(\vec{r}\,' \mid \vec{r}; \omega) \cdot \vec{k}^{(0)}_{mq}(\vec{r})\, dS = 0 \qquad m = 1, 2, \cdots, N. \tag{5}$$

The coupled-mode perturbation approximation is to assume that the current distributions of the coupled system modes are similar to the corresponding isolated mode currents, for loose, nearly degenerate coupling, i.e.,

$$\vec{k}_{nq} \approx a_{nq} \vec{k}^{(0)}_{nq} \qquad n = 1, 2, \cdots, N. \tag{6}$$

This assumption has been used in a similar manner to quantify the propagation eigenvalues of coupled microstrip transmission lines [9]. With current (6), the set of system mode equations (5) become

$$\sum_{n=1}^{N} C^q_{mn}(\omega) a_{nq} = 0 \qquad m = 1, 2, \cdots, N \tag{7}$$

with non-trivial solutions for current amplitudes a_{nq} only for complex natural frequencies $\omega = \omega_q$ determined from $\det[C^q_{mn}(\omega)] = 0$. Coupling coefficients C^q_{mn} are identified as

$$C^q_{mn}(\omega) = \int_{S_n} dS'\, \vec{k}^{(0)}_{nq}(\vec{r}\,') \cdot \int_{S_m} \overleftrightarrow{G}^e(\vec{r}\,' \mid \vec{r}; \omega) \cdot \vec{k}^{(0)}_{mq}(\vec{r})\, dS. \tag{8}$$

For nearly-identical disks, the operating frequency regime of significant interaction between disks is identified as $\omega \approx \omega^{(0)}_{mq} \approx \omega^{(0)}_{nq}$. A Taylor's series expansion of $\overleftrightarrow{G}^e$ about $\omega^{(0)}_{mq}$ is consequently prompted, leading to

$$C^q_{mn}(\omega) = \int_{S_n} dS'\, \vec{k}^{(0)}_{nq}(\vec{r}\,') \cdot \int_{S_m} \left[\overleftrightarrow{G}^e(\vec{r}\,' \mid \vec{r}; \omega^{(0)}_{mq}) + \left.\frac{\partial \overleftrightarrow{G}^e}{\partial \omega}\right|_{\omega^{(0)}_{mq}} (\omega - \omega^{(0)}_{mq}) + \cdots \right] \cdot \vec{k}^{(0)}_{mq}(\vec{r})\, dS. \tag{9}$$

The leading term vanishes for $n = m$, by (4) for the resonant current on the mth isolated disk. The coupling coefficient for the $n = m$ term becomes

$$C^q_{mm}(\omega) \approx \overline{C}^q_{mm}[\omega - \omega^{(0)}_{mq}] \tag{10}$$

where, by reciprocity of $\overleftrightarrow{G}^e$,

$$\overline{C}^q_{mm} \approx \int_{S_m} dS\, \vec{k}^{(0)}_{mq}(\vec{r}) \cdot \int_{S_m} \left.\frac{\partial \overleftrightarrow{G}^e(\vec{r} \mid \vec{r}\,'; \omega)}{\partial \omega}\right|_{\omega^{(0)}_{mq}} \cdot \vec{k}^{(0)}_{mq}(\vec{r}\,')\, dS'. \tag{11}$$

When $n \neq m$, the leading term in (9) is non-vanishing. The term proportional to $(\omega - \omega^{(0)}_{mq})$ is consequently rendered second-order small, leading to

$$C^q_{mn} \approx \int_{S_n} dS'\, \vec{k}^{(0)}_{nq}(\vec{r}\,') \cdot \int_{S_m} \overleftrightarrow{G}^e(\vec{r}\,' \mid \vec{r}; \omega^{(0)}_{mq}) \cdot \vec{k}^{(0)}_{mq}(\vec{r})\, dS \qquad (n \neq m). \tag{12}$$

Exploiting Equations (10) and (12) in the system mode equations (7) leads to the coupled-mode perturbation equations:

$$[\omega - \omega_{mq}^{(0)}]a_{mq}\overline{C}_{mm}^{q} + \sum_{n \neq m} C_{mn}^{q} a_{nq} = 0 \qquad \cdots m = 1, 2, \cdots, N \tag{13}$$

which depend only upon the frequency independent coupling coefficients (11) and (12). These constants are determined by the isolated resonant frequency $\omega_{mq}^{(0)}$ and isolated currents $\vec{k}_{mq}^{(0)}$, which are obtained by a full-wave MoM solution of the EFIE for an isolated disk, (4).

Natural system modes are obtained from the solution of det $[C_{mm}^{q}(\omega)] = 0$ which leads to nontrivial solutions for amplitudes a_{nq}. For the case of two nearly-degenerate disks, the system mode frequency is obtained from the solution of

$$\det \begin{bmatrix} (\omega - \omega_{1q}^{(0)})\overline{C}_{11}^{q} & C_{12}^{q} \\ C_{21}^{q} & (\omega - \omega_{2a}^{(0)})\overline{C}_{22}^{q} \end{bmatrix} = 0 \tag{14}$$

as $\omega = \overline{\omega} \pm \delta$, where $\overline{\omega} = (\omega_1 + \omega_2)/2$ is the average of the two isolated resonant frequencies and $\delta = \sqrt{\Delta^2 + k^2}$ with $\Delta = (\omega_1 - \omega_2)/2$ and $k^2 = C_{12}^{q}C_{21}^{q}/(\overline{C}_{11}^{q}\overline{C}_{22}^{q})$. The ratio of natural-mode currents is found as

$$\frac{a_2}{a_1} = \frac{\Delta}{C_{12}} \mp \sqrt{\left(\frac{\Delta}{C_{12}}\right)^2 + \frac{C_{21}}{C_{12}}}. \tag{15}$$

These modes correspond to π (antisymmetric) or c (symmetric) modes for the general case of non-identical elements. For two identical elements, $\omega_1 = \omega_2 = \omega_0$ and $\overline{C}_{11}^{q} = \overline{C}_{22}^{q}$ such that $\omega = \omega_0 \pm C_{12}$ and $a_2 = \mp a_1$. These system modes correspond to odd (antisymmetric)/even (symmetric) coupling between the two system elements.

IV. MoM Solution for Coupled Disks with Entire Domain Basis Functions

In this section, the coupled set of EFIE's (3) is solved by an entire domain basis function (EBF) MoM solution, to provide a comparison to the approximate coupled-mode theory presented in the previous section.

The current on the nth disk is expanded in a set of EBF's

$$\vec{K}_n(\vec{r}) = \sum_{j=1}^{J} a_{nj}\vec{K}_{nj}(\vec{r}) \qquad \cdots n = 1, 2, \cdots, N \tag{16}$$

where $\vec{K}_{nj}(\vec{r})$ are the expansion functions for the nth disk and a_{nj} are the corresponding weighting coefficients. EFIE's (3) then become

$$\hat{t}_m \cdot \sum_{n=1}^{N} \int_{S_n} \vec{\vec{G}}^{e}(\vec{r} \mid \vec{r}\,') \cdot \sum_{j=1}^{J} a_{nj}\vec{K}_{nj}(\vec{r}\,')\, dS' = 0$$

$$\forall \vec{r} \in S_m \qquad m = 1, 2, \cdots, N.$$

Assuming that each disk has the same number of current elements, J, the above system has N equations and NJ unknowns. Testing with the operator

$$\int_{S_m} dS\, \vec{K}_{ml}(\vec{r}) \cdot \qquad \cdots \begin{cases} m = 1, 2, \cdots, N \\ l = 1, 2, \cdots, J \end{cases}$$

which replaces the $(\hat{t}_m \cdot)$ operation leads to the $(JN) \times (JN)$ system of equations

$$\sum_{n=1}^{N} \sum_{j=1}^{J} a_{nj} \int_{S_m} dS\, \vec{K}_{ml}(\vec{r}) \cdot \int_{S_n} \vec{\vec{G}}^{e}(\vec{r} \mid \vec{r}\,') \cdot \vec{K}_{nj}(\vec{r}\,')\, dS' = 0 \qquad \cdots \begin{cases} m = 1, 2, \cdots, N \\ l = 1, 2, \cdots, J. \end{cases} \tag{17}$$

The solution of the coupled set of equations (17) leads to system eigenmodes, obtained with a numerical root-search.

V. Numerical and Experimental Results for Coupled Disks

In this section, numerical results using the perturbation theory, (13), are compared with measurements and with (17), the full-wave MoM solution of the coupled set of EFIE's (3). The perturbation theory utilizes the MoM solution of EFIE (4) for the eigenmodes of an isolated disk, which was solved assuming longitudinal current only. This has been found to be valid for rectangular disks having width much less than a half-wavelength in the film region [7].

The isolated EFIE (4) was solved using EBF's

$$K_{nj}(\vec{r}) = \frac{\cos\left[\dfrac{j\pi z}{2l}\right]}{\sqrt{1 - \left[\dfrac{x}{w}\right]^2}} \qquad \begin{matrix} j = 1, 3, 5, \cdots, J \\ n = 1, 2, \cdots, N \end{matrix} \tag{18}$$

for even modes, or

$$K_{nj}(\vec{r}) = \frac{\sin\left[j\pi \dfrac{z}{l}\right]}{\sqrt{1 - \left[\dfrac{x}{w}\right]^2}} \qquad \begin{matrix} j = 1, 2, 3, \cdots, J \\ n = 1, 2, \cdots, N \end{matrix} \tag{19}$$

for odd modes, which were also used to solve (17). These expansion functions were chosen to model the current behavior near resonance and are found to be similar to those used in [1]. A convergence study was performed to determine how many expansion functions were needed to adequately model the eigenmode current. Table I shows the results of a root search for the resonant wavenumber of an isolated disk in the cover region, $k_r^{(0)}l$, where a_j is the normalized amplitude coefficient of the jth term and l is the disk half-length. The change in the resonant wavenumber from $J = 1$ to $J = 3$ terms is less than 0.06%, and so typically only one expansion function is required to model the current near resonance, in agreement with

TABLE I
ISOLATED DISK RESONANT WAVENUMBERS FOR J TERM EBF SOLUTION

J	$k_r^{(0)} l$	a_1	a_2	a_3
1	(1.14196, .000801)	1.0	0.0	0.0
2	(1.14161, .000803)	1.0	0.0091	0.0
3	(1.14131, .000804)	1.0	0.0091	0.0002

l = 2.5 cm, w = .0784 cm, t = .0787 cm, ϵ_f = (2.20, −.002)

[10], [11]. It has been found that as the thickness of the film layer increases, the coefficients of the second and third terms in the current expansion increase, although they are still small compared to that of the first term and may be neglected.

Experiments have been performed on the system of coupled disks shown in Fig. 1. Two $2l$ = 5.0 cm disks of width $2w$ = .1568 cm are separated along the x axis by $2d_1$, measured from center to center as shown in Fig. 2. The center to center disk separation along the z axis is given by d_2. For generality, one of the disks is allowed to rotate according to the angle θ as shown in Fig. 1. The dielectric film was RT/Duroid with $\epsilon_f = (2.20, -j.002)$ and t = .0787 cm. The experiment consisted of measuring the port-to-port transmission between two probes loosely coupled to the microstrip disks, utilizing the swept frequency capabilities of a network analyzer [12]. Peaks of transmission indicate the location of system resonances.

Fig. 3 shows the real resonant system wavenumber in the cover region, k_r, for identical, parallel coupled disks ($d_2 = 0$, $\theta = 0$) as transverse separation d_1 varies. The solid line is the MoM solution and the dashed line is the perturbation result. As the transverse separation increases, the system wavenumbers tend towards the isolated disk limit, $k_r^{(0)}$. As the disks are brought together, the system modes emerge as symmetric and antisymmetric modes. Comparing results obtained by the various methods, it is seen that the perturbation approximation is in good agreement with the MoM solution and with experiment for loosely coupled and moderately coupled disks. The perturbation method is found to be valid for transverse separation distances up to approximately 60% of the disk width. It should also be noted that the symmetric mode solution of the perturbation approximation is in relatively good agreement with the MoM solution and experiment for tight coupling. The antisymmetric mode arises from a more complicated interaction between disks, and so even the MoM solution for the antisymmetric mode begins to lose accuracy for tight coupling. This could be improved by increasing the number of expansion functions, although this was not pursued here.

Fig. 4 shows the system wavenumbers for two identical coupled disks as the longitudinal separation d_2 varies. For this configuration d_1 = 0.1 cm and θ = 0. The system modes again form symmetric and antisymmetric modes, with maximum mode separation occurring when the disks overlap over half their length.

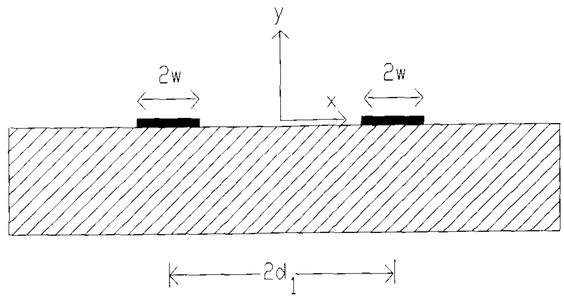

Fig. 2. Two coupled disks with transverse separation $2d_1$.

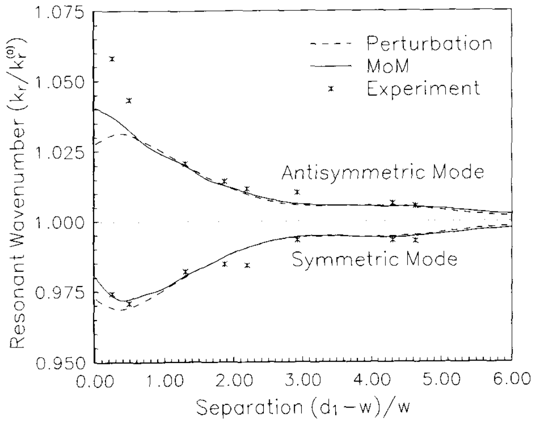

Fig. 3. Comparison of perturbation-theory-predicted, MoM predicted, and experimentally measured system wavenumbers for two identical coupled microstrip disks as their transverse separation is varied. ϵ_f = 2.20, t = .0787 cm, l = 2.5 cm, w = .0784 cm, d_2 = 0.0 cm, θ = 0.0°

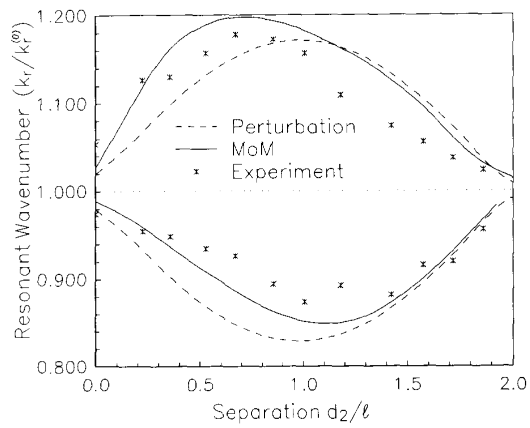

Fig. 4. Comparison of perturbation-theory-predicted, MoM predicted, and experimentally measured system wavenumbers for two identical coupled microstrip disks as their longitudinal separation is varied. ϵ_f = 2.20, t = .0787 cm, l = 2.5 cm, w = .0784 cm, d_1 = 0.1 cm, θ = 0.0°

The system resonant wavenumbers of two identical coupled disks are shown in Fig. 5, as the angle between them varies. The longitudinal displacement is d_2 = 2.6 cm, and the transverse separation is d_1 = 0.1 cm. The relative angle between the disks, θ, is varied from 0 to 40

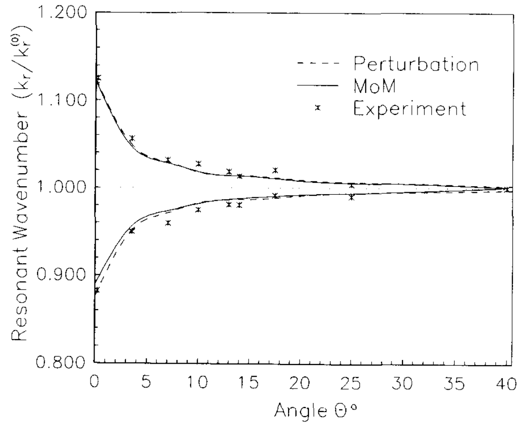

Fig. 5. Comparison of perturbation-theory-predicted, MoM predicted, and experimentally measured system wavenumbers for two identical coupled microstrip disks as the angle between them is varied. $\epsilon_r = 2.20$, $t = .0787$ cm, $l = 2.5$ cm, $w = .0784$ cm, $d_1 = 0.1$ cm, $d_2 = 2.6$ cm

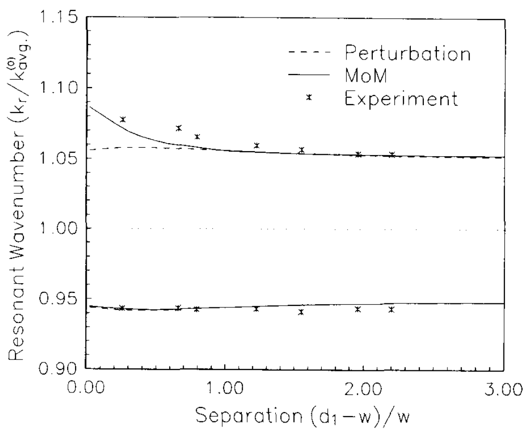

Fig. 6. Comparison of perturbation-theory-predicted, MoM predicted, and experimentally measured system wavenumbers for two non-identical coupled microstrip disks as their transverse separation is varied. $\epsilon_r = 2.20$, $t = .0787$ cm, $l_1 = 2.5$ cm, $l_2 = 2.25$ cm, $w_1 = w_2 = .0784$ cm, $d_2 = 0.0$ cm, $\theta = 0.0°$

degrees. It can be seen that the maximum coupling exists between disks when $\theta = 0$ degrees, and that the coupling decreases as θ increases until the disks are virtually uncoupled.

It should be noted that in all of the above figures, results are presented for system frequencies near the dominant mode, $q = 1$ in (3). The theoretical curves were normalized to the same isolated resonant wavenumber, obtained from (4), and the experimental points were normalized to the measured isolated resonant wavenumber. These isolated wavenumbers differed by less than 1.5%.

The system of two coupled, non-identical disks is considered in Fig. 6, for $2l_1 = 5.0$ cm, $2l_2 = 4.5$ cm, and $2w_1 = 2w_2 = .1568$ cm. The system wavenumber is shown as the transverse separation, d_1, varies, with $d_2 = 0$, $\theta = 0$. For this system, the two modes split symmetrically about the average of the isolated disks' resonant wavenumbers, $k_{avg.}^{(0)} = (k_{r1}^{(0)} + k_{r2}^{(0)})/2$, which is used to normalize the plot.

TABLE II
COMPARISON OF COST OF MoM AND PERTURBATION SOLUTIONS

N	M	Cost_{MoM}	$\text{Cost}_{\text{pert.}}$	Cost Ratio
2	1	30	13	2.3
3	1	90	21	4.3
4	1	200	30	6.7
2	10	300	22	13.6
3	10	900	48	18.8
4	10	2000	84	23.8

VI. Discussion

The perturbation approximation is found to be an efficient method of determining the system eigenmodes of coupled microstrip disks, compared to the full MoM solution which requires a numerical root search of the coupled system of equations (17). As an example, consider an N-element system of non-identical coupled disks, each with one expansion function per element. The evaluation of one "Sommerfeld-type" integral will be considered as one unit of "cost."

The full MoM solution requires $N(N + 1)/2$ units to fill the matrix obtained from (17). The root-search requires approximately five iterations for each of the N independent modes, such that the full MoM solution costs $2.5N^2(N + 1)$ units. To evaluate the system eigenmodes for M different geometries (different values of d_1, d_2, θ) requires a total cost of $2.5N^2M(N + 1)$ units.

The perturbation theory requires a root-search for the eigenmodes of each isolated element, based on (4). Five iterations for the each of the N different disks is required once, with the addition of the evaluation of (11) N times. A total of $N(N - 1)/2$ evaluations of coupling coefficient (12) is required for each of the M different geometries, bringing the total cost to $6N + MN(N - 1)/2$ units. The cost ratio, defined as $\text{cost}_{\text{MoM}}/\text{cost}_{\text{pert.}}$ is given in Table II. It is found that the perturbation theory is at least twice as fast as the MoM solution for the worst case scenario of two coupled disks when only one geometrical configuration is of interest. The time savings increase as the number of coupled disks increase or the number of different geometries of interest increase.

VII. Conclusion

A full-wave perturbation theory for the analysis of coupled microstrip disks has been developed. This method is based on the system of coupled EFIE's which rigorously model the microstrip environment. The perturbation theory makes use of the isolated disks' current distributions and resonant frequencies to characterize the coupled system of disks, and is found to be very efficient compared to the full MoM solution to the problem. For the case of two coupled disks, the system eigenmodes are seen to correspond to symmetric and antisymmetric modes, shifted symmetrically about their isolated limits. Numerically, the perturbation theory agrees with the full MoM solution and with experiment for loose to moderate coupling.

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