Full-Wave Perturbation Theory for the Analysis of Coupled Microstrip Resonant Structures

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Abstract—A full-wave perturbation theory for the system of \( N \) coupled microstrip disk structures is presented. The theory is based on the electric field integral equation description of the circuit, which includes all of the wave phenomena associated with the conductors and the surrounding media. This method is suitable for quantification of nearly degenerate coupling between open microstrip disks, yielding the complex system eigenmodes. For the case of two coupled disks, the perturbation theory analytically separates, though simultaneously solves for, the symmetric and antisymmetric system eigenmodes. The development of the perturbation theory leads to good physical insight for this mode splitting phenomena. Numerical results obtained with the perturbation theory agree well with those obtained by a more accurate method of moments solution to the coupled set of electric field integral equations, as well as with experimental data.

I. INTRODUCTION

PRINTED microstrip disk structures are used extensively as microwave and millimeter-wave integrated circuit components. A number of theoretical methods have been developed to analyze the resonant characteristics of these structures, the most accurate techniques arising from rigorous full-wave spectral or space domain methods which account for all wave phenomena associated with the structures and the surrounding media [1]–[7]. Application of these methods for the study of system resonances of coupled microstrip elements is quite involved both analytically and numerically, and becomes prohibitive as the number of elements increases.

In this paper, a full-wave perturbation theory for the analysis of the eigenmodes of the system of \( N \) coupled microstrip disk structures in an open (unshielded) environment is developed, based upon a plane-wave spectral domain formulation of coupled electric field integral equations (EFIE’s). The coupled set of EFIE’s are solved in an efficient manner by approximating the system eigenvalue equation and implementation of the perturbation theory leads to good physical insight for this mode splitting phenomena. Numerical results obtained with the perturbation theory agree well with those obtained by a more accurate method of moments solution to the coupled set of electric field integral equations, as well as with experimental data.

The configuration of coupled microstrip disks is shown in Fig. 1 for \( N = 2 \) narrow rectangular structures, although this procedure may be applied to other printed disk shapes. The conducting disks are located in the cover region of the tri-layered conductor/film/cover environment, at the film/cover interface. The coordinate origin is chosen at the film/cover interface, with \( y \) normal to and \( z \) tangential to that interface. The cover and film regions are characterized by \( \epsilon_i = n_i^2 \epsilon_0 \) and \( \mu_i = \mu_0 \) for \( i = f, c \), where \( n_i \) is the refractive index of the \( i \)th layer. The wavenumber and intrinsic impedance of each layer are \( k_i = n_i k_0 \) and \( \eta_i = \eta_0 / n_i \) where \( (k_0, \eta_0) \) are their free-space counterparts. In this work, the cover region is assumed to be free space.

The electric field in the cover is obtained from the Hertzian potential there [8], and implementation of the boundary condition for tangential electric fields at the conducting disk’s surfaces leads to the coupled set of EFIE’s for surface current \( \vec{K}_m \) induced on the \( m \)th disk by impressed excitation \( \vec{E}^i \) as

\[
\vec{E}_m \cdot \sum_{n=1}^{N} \int_{S_n} \vec{G}(\vec{r} | \vec{r}') \cdot \vec{K}_n(\vec{r}') dS' = -\frac{jk_m}{\eta_c} \vec{E}_m \cdot \vec{E}^i(\vec{r}) \quad \forall \vec{r} \in S_m
\]

\( m = 1, 2, \cdots, N \) (1)

where \( \vec{E}_m \) is a unit vector tangent to the \( m \)th disk and \( \vec{r} = \vec{r} + \hat{x} + \hat{y} + \hat{z} \) is the 3-D position vector. The electric dyadic Green’s function is given in [8], and will not be repeated here. The coupled set of EFIE’s (1) provide the fundamental resource for the investigation of EM phenomena in multi-disk systems.

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The coupled-mode perturbation approximation is to assume that the current distributions of the coupled system modes are similar to the corresponding isolated mode currents, for loose, nearly degenerate coupling, i.e.,

$$k_{nq} = a_{nq} k_{nq}^{(0)} \quad n = 1, 2, \ldots, N.$$  (6)

This assumption has been used in a similar manner to quantify the propagation eigenvalues of coupled microstrip transmission lines [9]. With current (6), the set of system mode equations (5) become

$$\sum_{n=1}^{N} C_{mn}^{q}(\omega) a_{nq} = 0 \quad m = 1, 2, \ldots, N$$  (7)

with non-trivial solutions for current amplitudes $a_{nq}$ only for complex natural frequencies $\omega = \omega_q$ determined from $\det[C_{mn}(\omega)] = 0$. Coupling coefficients $C_{mn}^{q}$ are identified as

$$C_{mn}^{q}(\omega) = \int_{S_m} dS' \overline{k}_{nq}^{(0)}(\vec{r}') \cdot \int_{S_m} \overline{G}'(\vec{r}' \mid \vec{r}; \omega) \cdot k_{m'}^{(0)}(\vec{r}) dS.$$  (8)

For nearly-identical disks, the operating frequency regime of significant interaction between disks is identified as $\omega = \omega_{mq} \approx \omega_{nq}^{(0)}$. A Taylor's series expansion of $\overline{G}'$ about $\omega_{mq}$ is consequently prompted, leading to

$$C_{mn}^{q}(\omega) = \int_{S_m} dS' \overline{k}_{nq}^{(0)}(\vec{r}') \cdot \int_{S_m} \frac{\partial \overline{G}'(\vec{r}' \mid \vec{r}; \omega)}{\partial \omega} \bigg|_{\omega_{mq}}^{(0)} + \cdots$$

$$\cdot \overline{k}_{m'}^{(0)}(\vec{r}) dS.$$  (9)

The leading term vanishes for $n = m$, by (4) for the resonant current on the $m$th isolated disk. The coupling coefficient for the $n = m$ term becomes

$$C_{mn}^{q}(\omega) = \overline{C}_{mn}^{q}[\omega - \omega_{mq}]$$  (10)

where, by reciprocity of $\overline{G}'$,

$$\overline{C}_{mn}^{q} = \int_{S_m} dS' \overline{k}_{nq}^{(0)}(\vec{r}') \cdot \int_{S_m} \frac{\partial \overline{G}'(\vec{r}' \mid \vec{r}; \omega)}{\partial \omega} \bigg|_{\omega_{mq}}^{(0)}$$

$$\cdot \overline{k}_{m'}^{(0)}(\vec{r}) dS'.$$  (11)

When $n \neq m$, the leading term in (9) is non-vanishing. The term proportional to $(\omega - \omega_{mq})$ is consequently rendered second-order small, leading to

$$C_{mn}^{q} = \int_{S_m} dS' \overline{k}_{nq}^{(0)}(\vec{r}') \cdot \int_{S_m} \overline{G}'(\vec{r}' \mid \vec{r}; \omega_{mq})$$

$$\cdot \overline{k}_{m'}^{(0)}(\vec{r}) dS \quad (n \neq m).$$  (12)

Exploiting Equations (10) and (12) in the system mode equations (7) leads to the coupled-mode perturbation
equations:
\[
[\omega - \omega_m^{(0)}] \mathbf{C}_{mm} + \sum_{n \neq m} C_{mn}^{\mathbf{q}} a_{\mathbf{q}} = 0
\]
\[
\cdots m = 1, 2, \ldots, N
\]  
(13)
which depend only upon the frequency independent coupling coefficients (11) and (12). These constants are determined by the isolated resonant frequency \( \omega_m^{(0)} \) and isolated currents \( \tilde{\mathbf{k}}_m^{(0)} \), which are obtained by a full-wave MoM solution of the EFIE for an isolated disk, (4).

Natural system modes are obtained from the solution of the operator
\[
\sum_{n \neq m} C_{mn}^{\mathbf{q}} a_{\mathbf{q}} = 0
\]
\[
\cdots m = 1, 2, \ldots, N
\]
which replaces the \( (f_m \cdot \cdot \cdot) \) operation leads to the \( (JN) \times (JN) \) system of equations
\[
\sum_{n=1}^{N} \sum_{j=1}^{J} a_{nj} \int_{S_m} dS \tilde{\mathbf{k}}_m^{(0)}(\vec{r}) \cdot \int_{S_m} \tilde{\mathbf{G}}(\vec{r} \cdot \vec{r}') = 0
\]
\[
\cdots \left\{ \begin{array}{c}
m = 1, 2, \ldots, N \\
l = 1, 2, \ldots, J
\end{array} \right.
\]
(14)

The solution of the coupled set of equations (17) leads to system eigenmodes, obtained with a numerical root-search.

V. NUMERICAL AND EXPERIMENTAL RESULTS FOR COUPLED DISKS

In this section, numerical results using the perturbation theory, (13), are compared with measurements and with (17), the full-wave MoM solution of the coupled set of EFIE’s (3). The perturbation theory utilizes the MoM solution of EFIE (4) for the eigenmodes of an isolated disk, which was solved assuming longitudinal current only. This has been found to be valid for rectangular disks having width much less than a half-wavelength in the film region [7].

The isolated EFIE (4) was solved using EBF’s

\[
K_{nj}(\vec{r}) = \frac{\cos \left[ \frac{j \pi x}{2l} \right]}{\sqrt{1 - \left( \frac{x}{w} \right)^2}} \quad j = 1, 3, 5, \ldots, J \\
\]  
(18)
for even modes, or
\[
K_{nj}(\vec{r}) = \frac{\sin \left[ \frac{j \pi x}{2l} \right]}{\sqrt{1 - \left( \frac{x}{w} \right)^2}} \quad j = 1, 2, 3, \ldots, J
\]
(19)
for odd modes, which were also used to solve (17). These expansion functions were chosen to model the current behavior near resonance and are found to be similar to those used in [1]. A convergence study was performed to determine how many expansion functions were needed to adequately model the eigenmode current. Table I shows the results of a root search for the resonant wavenumber of an isolated disk in the cover region, \( k_{\text{eff}}^{(0)} \), where \( a_j \) is the normalized amplitude coefficient of the \( j \)th term and \( l \) is the disk half-length. The change in the resonant wavenumber from \( J = 1 \) to \( J = 3 \) terms is less than 0.06%, and so typically only one expansion function is required to model the current near resonance, in agreement with
It has been found that as the thickness of the film layer increases, the coefficients of the second and third terms in the current expansion increase, although they are still small compared to that of the first term and may be neglected.

Experiments have been performed on the system of coupled disks shown in Fig. 1. Two \( 2t = 5.0 \) cm disks of width \( 2w = 0.1568 \) cm are separated along the \( x \) axis by \( 2d_1 \), measured from center to center as shown in Fig. 2. The center to center disk separation along the \( z \) axis is given by \( d_2 \). For generality, one of the disks is allowed to rotate according to the angle \( \theta \) as shown in Fig. 1. The dielectric film was RT/Duroid with \( \epsilon_r = (2.20, -j0.02) \) and \( t = 0.0787 \) cm. The experiment consisted of measuring the port-to-port transmission between two probes loosely coupled to the microstrip disks, utilizing the swept frequency capabilities of a network analyzer [12]. Peaks of transmission indicate the location of system resonances.

Fig. 3 shows the real resonant system wavenumber in the cover region, \( k_r \), for identical, parallel coupled disks \( (d_2 = 0, \theta = 0) \) as transverse separation \( d_1 \) varies. The solid line is the MoM solution and the dashed line is the perturbation result. As the transverse separation increases, the system wavenumbers tend towards the isolated disk limit, \( k_{0r} \). As the disks are brought together, the system modes emerge as symmetric and antisymmetric modes. Comparing results obtained by the various methods, it is seen that the perturbation approximation is in good agreement with the MoM solution and with experiment for loosely coupled and moderately coupled disks. The perturbation method is found to be valid for transverse separation distances up to approximately 60% of the disk width. It should also be noted that the symmetric mode solution of the perturbation approximation is in relatively good agreement with the MoM solution and experiment for tight coupling. The antisymmetric mode arises from a more complicated interaction between disks, and so even the MoM solution for the antisymmetric mode begins to lose accuracy for tight coupling. This could be improved by increasing the number of expansion functions, although this was not pursued here.

Fig. 4 shows the system wavenumbers for two identical coupled disks as the longitudinal separation \( d_2 \) varies. For this configuration \( d_1 = 0.1 \) cm and \( \theta = 0 \). The system modes again form symmetric and antisymmetric modes, with maximum mode separation occurring when the disks overlap over half their length.

The system resonant wavenumbers of two identical coupled disks are shown in Fig. 5, as the angle between them varies. The longitudinal displacement is \( d_1 = 2.6 \) cm, and the transverse separation is \( d_2 = 0.1 \) cm. The relative angle between the disks, \( \theta \), is varied from 0 to 40°.
degrees. It can be seen that the maximum coupling exists between disks when θ = 0 degrees, and that the coupling decreases as θ increases until the disks are virtually uncoupled.

It should be noted that in all of the above figures, results are presented for system frequencies near the dominant mode, q = 1 in (3). The theoretical curves were normalized to the same isolated resonant wavenumber, obtained from (4), and the experimental points were normalized to the measured isolated resonant wavenumber. These isolated wavenumbers differed by less than 1.5%.

The system of two coupled, non-identical disks is considered in Fig. 6, for 2l1 = 5.0 cm, 2l2 = 4.5 cm, and 2w1 = 2w2 = .1568 cm. The system wavenumber is shown as the transverse separation, d1, varies, with d2 = 0, θ = 0. For this system, the two modes split symmetrically about the average of the isolated disks’ resonant wavenumbers, k_0^w = (k_1^w + k_2^w)/2, which is used to normalize the plot.

### VI. Discussion

The perturbation approximation is found to be an efficient method of determining the system eigenmodes of coupled microstrip disks, compared to the full MoM solution which requires a numerical root search of the coupled system of equations (17). As an example, consider an N-element system of non-identical coupled disks, each with one expansion function per element. The evaluation of one “Sommerfeld-type” integral will be considered as one unit of “cost.”

The full MoM solution requires N(N + 1)/2 units to fill the matrix obtained from (17). The root-search requires approximately five iterations for each of the N independent modes, such that the full MoM solution costs 2.5N^2(N + 1) units. To evaluate the system eigenmodes for M different geometries (different values of d1, d2, θ) requires a total cost of 2.5M(N + 1) units.

The perturbation theory requires a root-search for the eigenmodes of each isolated element, based on (4). Five iterations for each of the N different disks is required once, with the addition of the evaluation of (11) N times. A total of N(N − 1)/2 evaluations of coupling coefficient (12) is required for each of the M different geometries, bringing the total cost to 6N + MN(N − 1)/2 units. The cost ratio, defined as cost_{MoM}/cost_{pert}, is given in Table II. It is found that the perturbation theory is at least twice as fast as the MoM solution for the worst case scenario of two coupled disks when only one geometrical configuration is of interest. The time savings increase as the number of coupled disks increase or the number of different geometries of interest increase.

### VII. Conclusion

A full-wave perturbation theory for the analysis of coupled microstrip disks has been developed. This method is based on the system of coupled EFIE’s which rigorously model the microstrip environment. The perturbation theory makes use of the isolated disks’ current distributions and resonant frequencies to characterize the coupled system of disks, and is found to be very efficient compared to the full MoM solution to the problem. For the case of two coupled disks, the system eigenmodes are seen to correspond to symmetric and antisymmetric modes, shifted symmetrically about their isolated limits. Numerically, the

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perturbation theory agrees with the full MoM solution and with experiment for loose to moderate coupling.

REFERENCES


